Prof. Dr. Gregor Schöner October 15, 2022

Computational Neuroscience: Neural Dynamics

Exercise 1, hand in by October 27, 2027

Use complete sentences where textual answers are requested. Explain symbols when using mathematical notation. In graphs that you provide, label the axes.

1. The linear dynamical system

$$\dot{x} = -\alpha x$$

governs the temporal evolution of a real-valued dynamical variable, x, where α is a parameter.

(a) Plot this equation for $\alpha > 0$ and $\alpha < 0$. Label the axes and write a short caption.

(b) Write down its solution as a formula and verify that this solution solves the equation by computing its derivative.

(c) Plot the solution for $\alpha > 0$ and at least two initial conditions. (A qualitative plot is sufficient, but you can also choose specific values for α and the initial values and do the plot numerically.)

(d) For $\alpha > 0$, compute the times, t_n, at which the solution reaches $x(0)/e^n$ (where $e = 2.7... = \exp(1)$). Compute $t_{n+1} - t_n$ (called "relaxation time" in physics). Does it depend on n? Explain your answer!

(e) Based on the last two results, how does the solution change as α becomes larger. Plot or describe.

2. The non-linear dynamical system

$$\dot{x} = a - x^2$$

governs the temporal evolution of a real-valued dynamical variable, x, where a is a parameter.

(a) Plot this equation for $a > 0$ and $a < 0$ and $a = 0$. Labeltheaxesandwrite a short caption.

(b) Determine the fixed points of this dynamics by solving for $\dot{x} = 0$.

(c) By "mental simulation" guess the asymptotic behavior when time goes to infinity for initial conditions below and above zero for $a < 0$ vs $a > 0$.

(d) At $a = 0$, solve the equation analytically. [Hint: use separation of variables leading to $\int_{x_0}^{x(t)} dx/x^2 = -t$ and solve the integral.]

(e) Examine what happens when time goes to infinity for $x_0 > 0$ and compare to your "mental simulation".

(f) Advanced question: More complex is to understand what happens for $x_0 < 0$. Can you explain that?