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Autonomous robotics

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Exercise 1 Dynamical Systems Tutorial

Please upload solutions on the web page before midnight on April 21 (Friday).

1. Solve the linear differential equation

$$\dot{x} = -\alpha x \tag{1}$$

analytically. If unfamiliar, get help from a text book or online resource.

2. Plot the solution as a function of time by setting $\alpha = 1$/second, using a few discrete times, $t = 0, 1, 2, ...10$ seconds. Or evaluate the function numerically on the computer in a program of your choice. Make this plot for two values of the initial condition, $x(0)$.

3. From the analytical solution, determine the moment in time, τ at which $x(t)$ has fallen to $1/e$ of its initial value, $x(0)$. Does this time depend on $x(0)$? Answer the same question for the hyperbolic decay, $x(t) = x(0)/(1 + t)$.

4. Plot this dynamics Eq (1) and designate the fixed point. Make the same drawing for a dynamics with the same slope and a fixed point at $x_0 > 0$. Write down the equation for that dynamics.

5. Make a plot of this nonlinear differential equation

$$\dot{x} = \beta - x^2. \tag{2}$$

Compute its fixed points as a function of β and plot this function. Mark the fixed points in the dynamics as attractors or repellors.