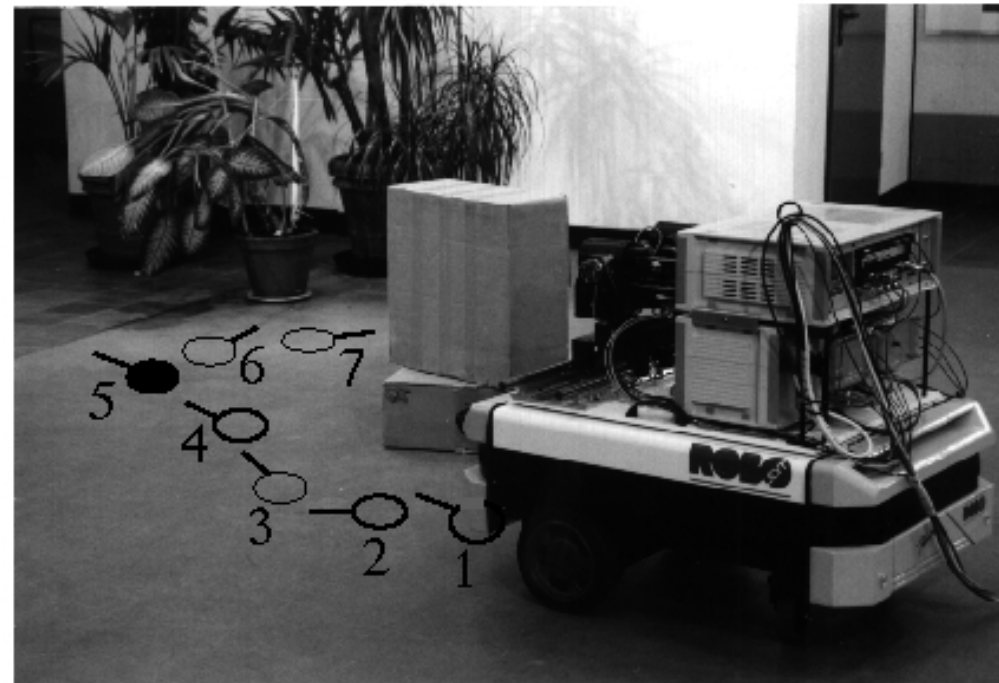


# Attractor dynamics approach to vehicle movement generation

Gregor Schöner, INI, RUB

# The vehicle movement problem

- move vehicle in a 2D world
- toward a target
- while avoiding collisions with obstacles
- potentially:
  - follow a road or a sequence of targets (via points)
  - docking: achieve a particular orientation

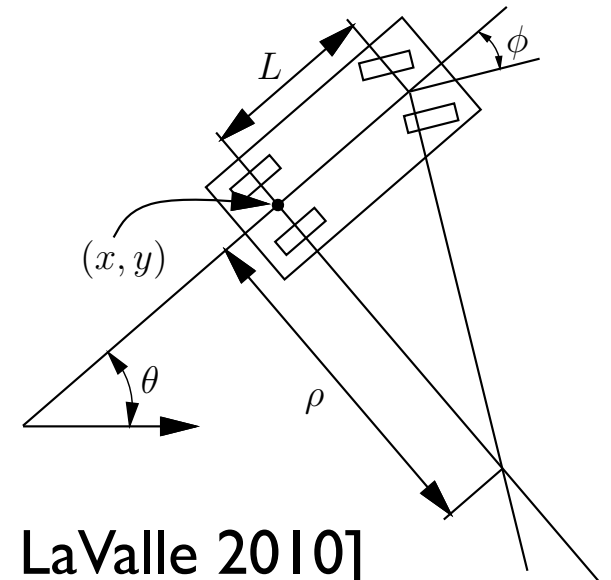
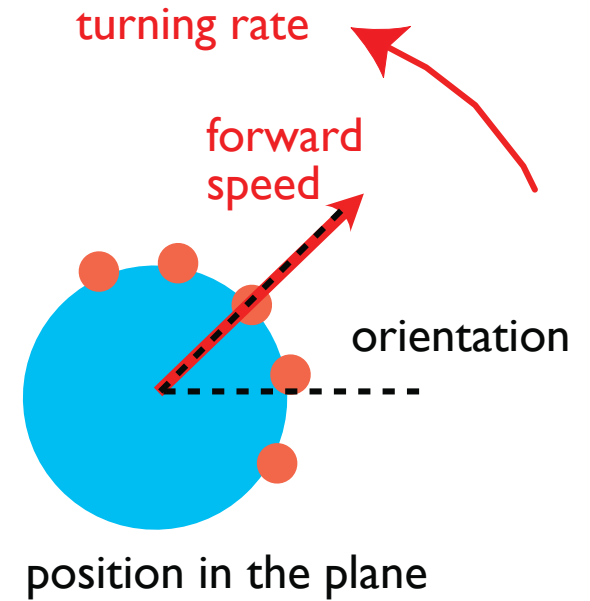


# Degree of Freedom (DoF)

■ Vehicles have 3 DoF

■ 2D position

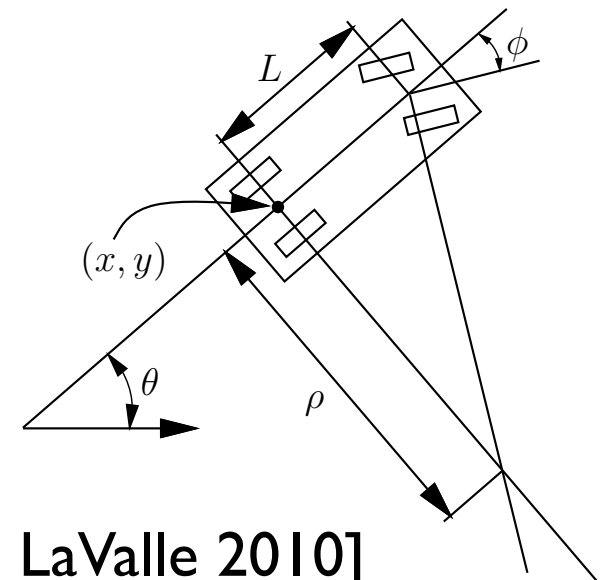
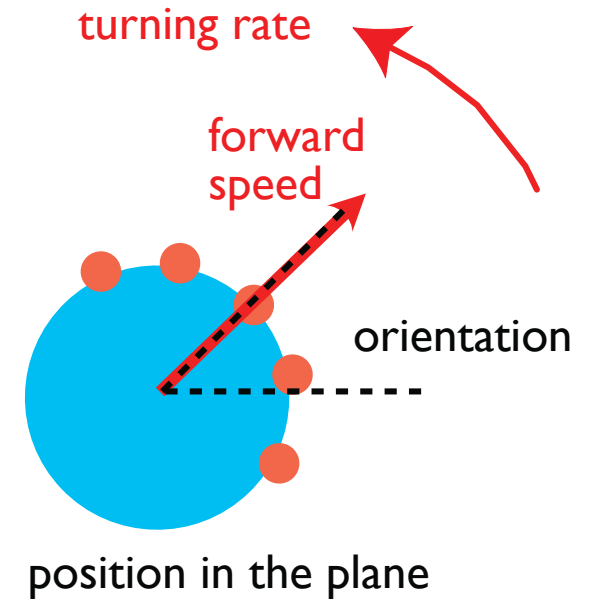
■ Orientation



[from LaValle 2010]

# Non-holonomic constraints

- fewer variables than the number of DoF can be varied freely
  - robot with two active wheels: 2 wheel velocities
  - car: steering angle and speed
- state of the 3DoF depends on the history of movement
  - easy for robot with active wheels: turn on the spot
  - difficult for car: parking



[from LaValle 2010]

# Autonomous vehicle movement

- sense something about the environment or know about the environment (map)
- plan movement in the environment toward target that is collision-free
- control the vehicle to achieve the planned movement
- estimate what vehicle actually did: update the map

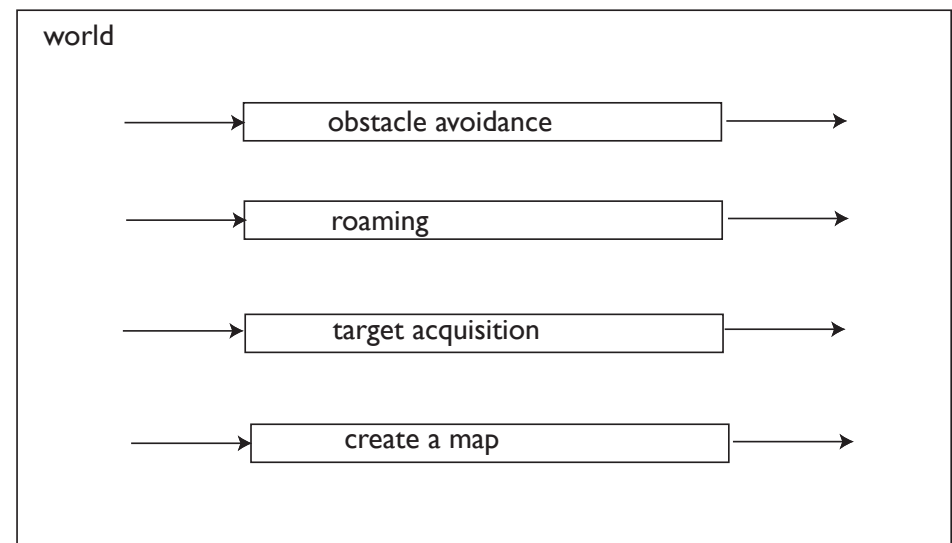
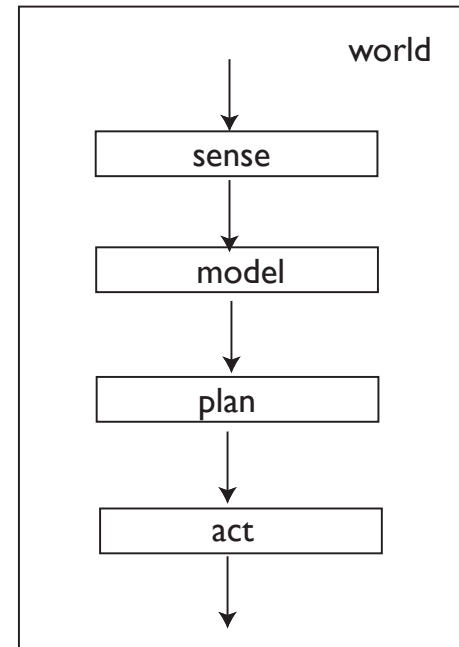
# Architectures

## ■ sense-plan-act

- planning based on a world world model

## ■ behavior-based

- low-level sensory information that is specific to each individual behavior
- planning emerges from how behaviors interact



# Concepts for planning

## ■ local vs. global

- planning based on information only about the local environment of the robot
- vs. based on global map information about the environment

## ■ reactive vs. planning

- motion planning “on the fly” in response to sensory inputs
- vs. motion planning for an entire action from initial to goal state

# Concepts for planning

## ■ exact vs. heuristic

- exact: guarantee that a path that fulfills the constraints is found when one exists
- vs. generate a plan based on ad hoc approach that is likely to fulfill constraints

## ■ continuous vs. discrete:

- continuous state space variables
- vs. grid state spaces, graph state spaces



# Attractor dynamics approach

- developed by my team over many years
- a particular solution to the vehicle motion planning problem that is conceptually compatible with properties of the nervous system and human/animal behavior
- it can be used both in sense-plan-act (“symbolic”) or behavior-based (“sub-symbolic”) form
- it is **local, reactive, heuristic, and continuous**

# Basic ideas of the attractor dynamics approach

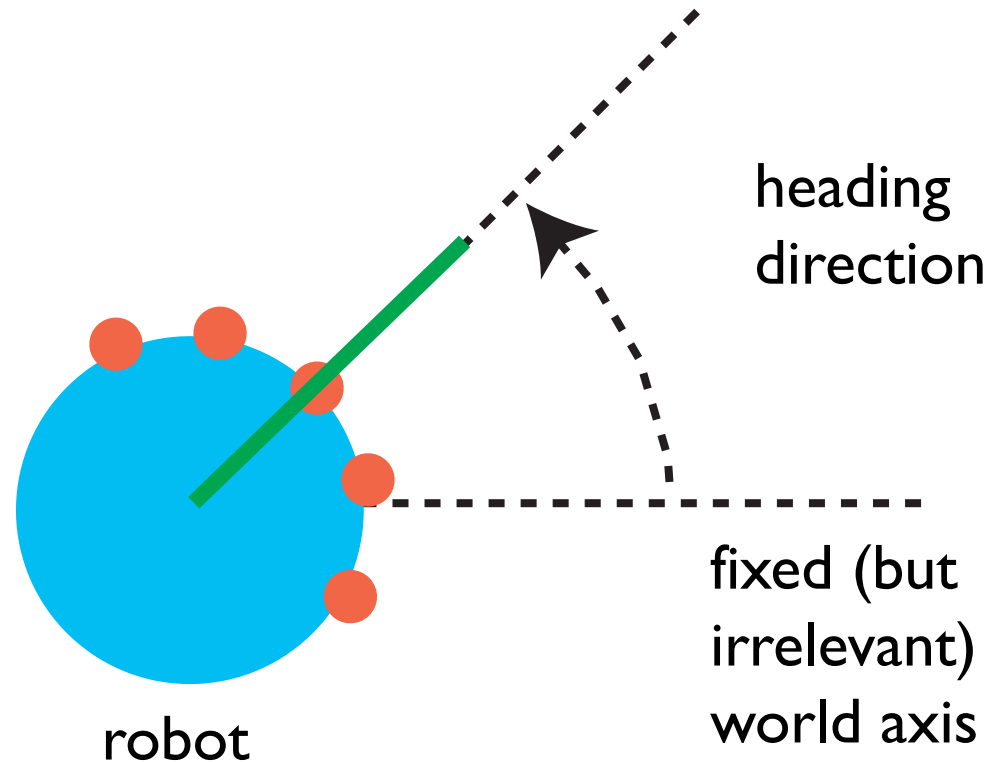
- plans are time courses of behavioral variables
- these time courses are generated by a dynamical system
- they are structured by attractor solutions of dynamical systems (which may change during the movement)
- decisions emerge from bifurcations of the attractor solutions

# Behavioral variables

■ first behavioral variable: heading direction

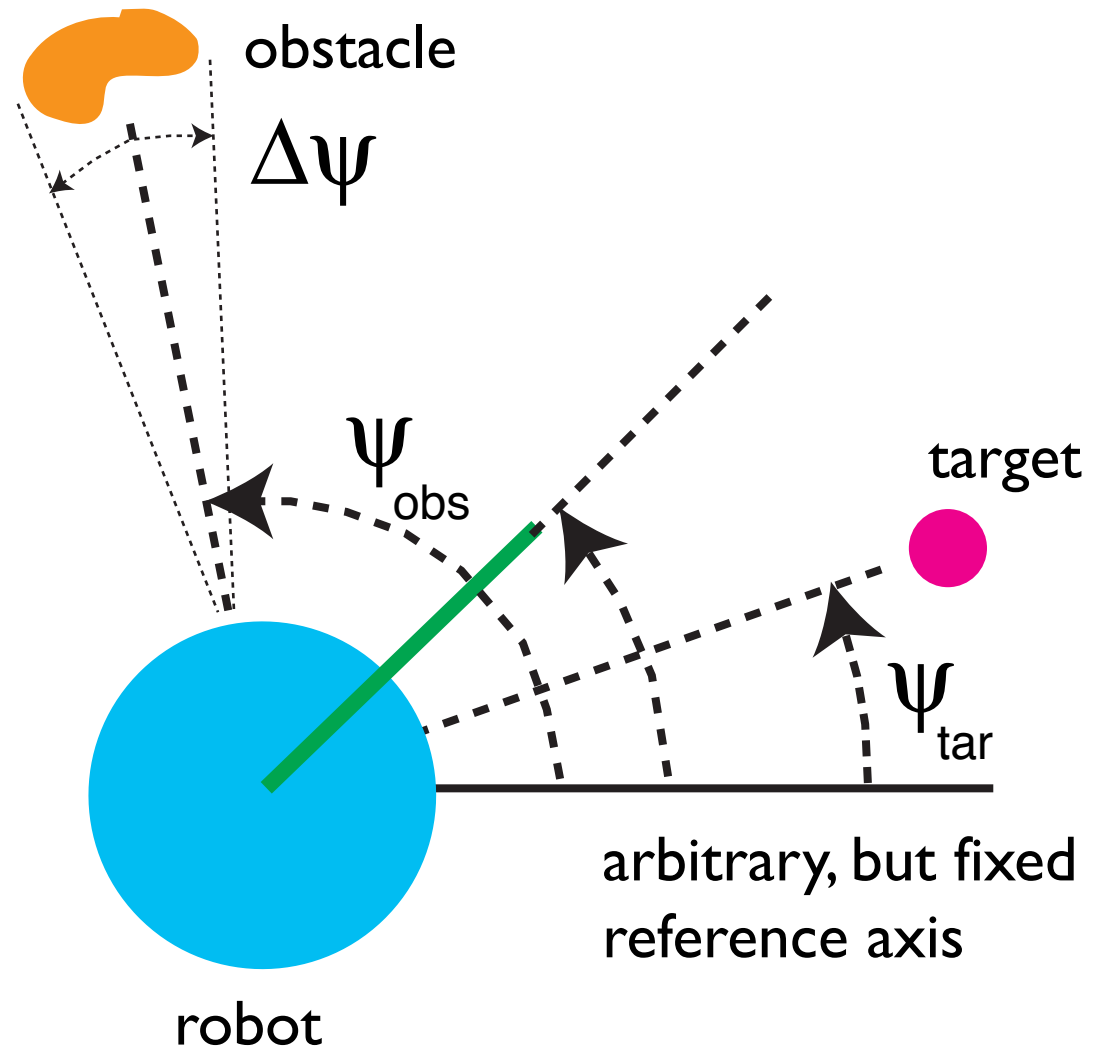
■ second behavioral variable: forward speed

■ (neglected in this lecture: constant speed)



# Constraints

- obstacle avoidance
- target acquisition



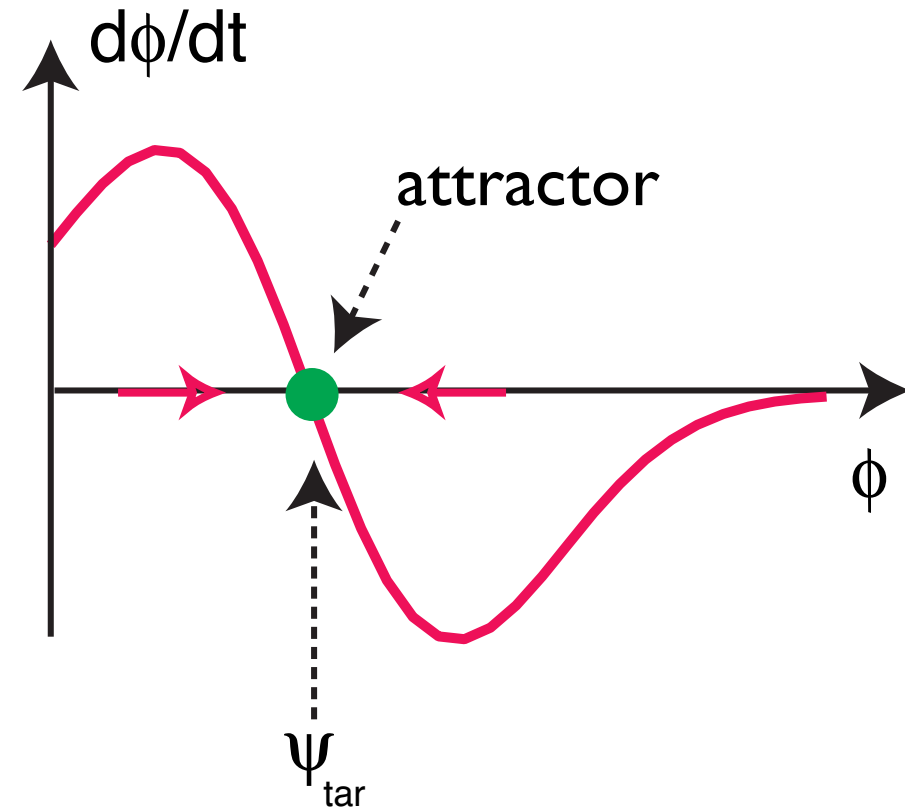
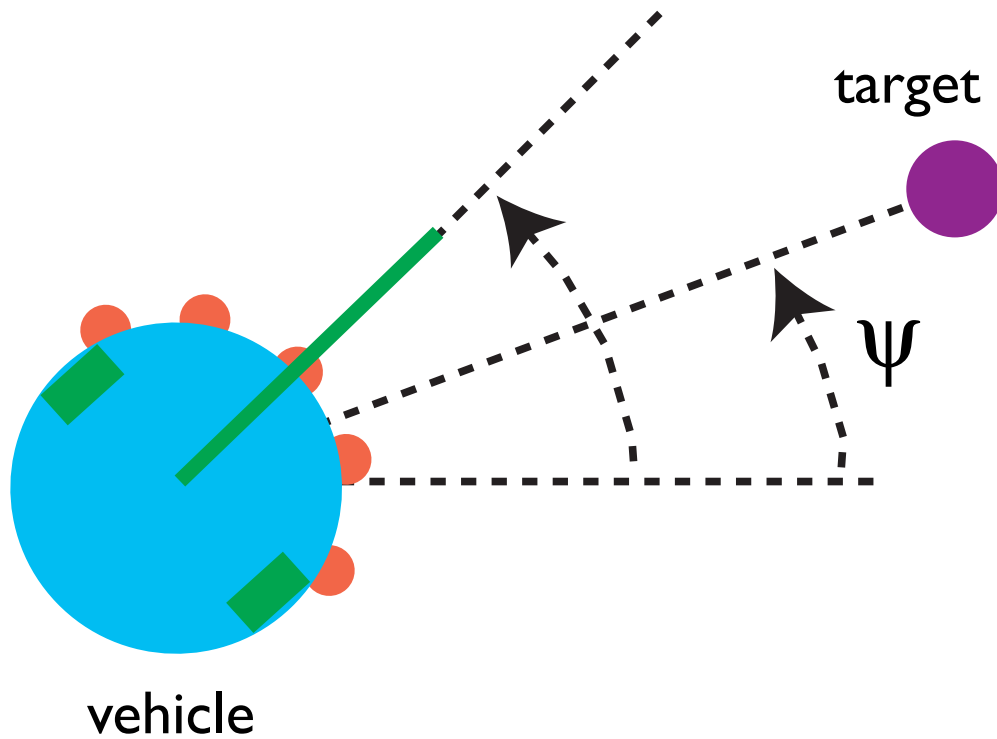
# Behavioral variables: properties

- uniquely describe the desired movement
- “enactable”: can be used to control the behavior
- constraints can be expressed as values/value ranges of the behavioral variables
- no calibration needed

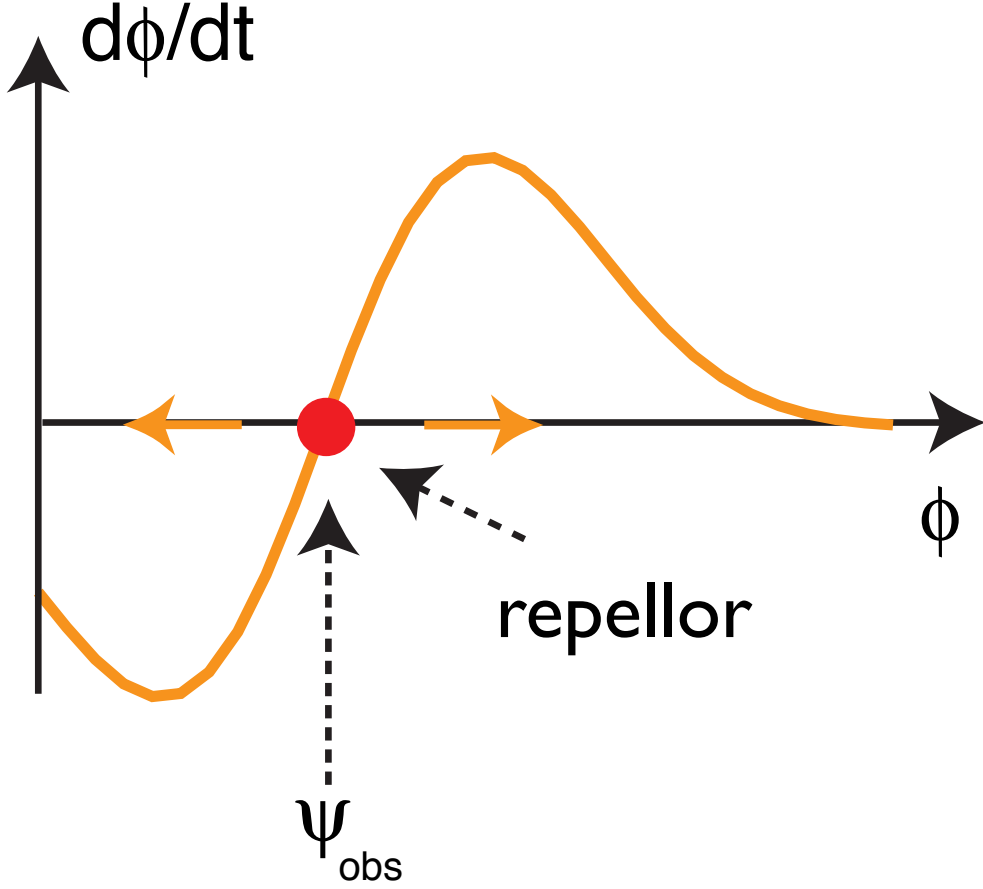
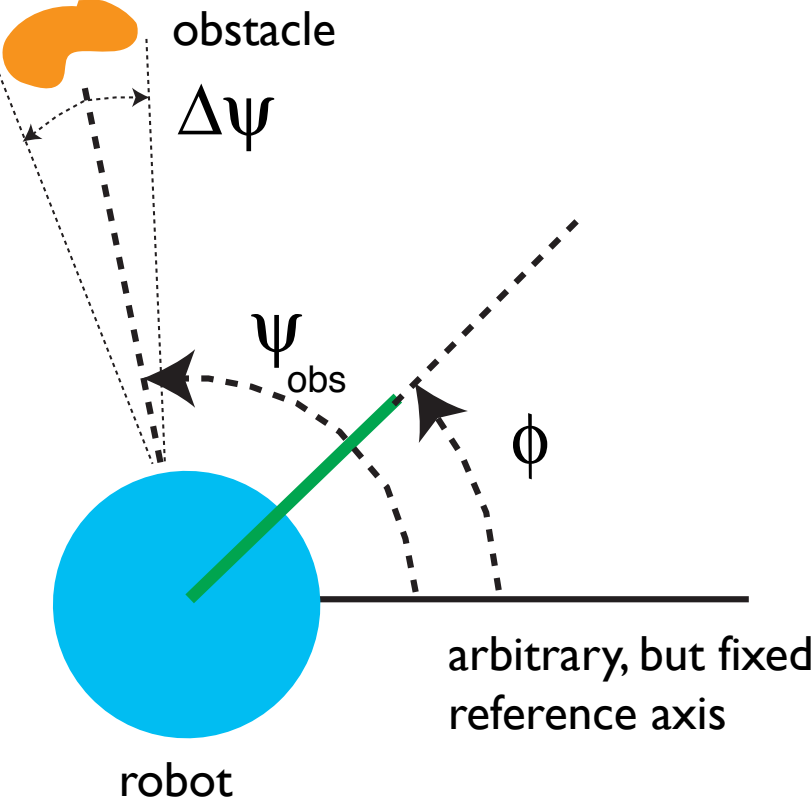
# Behavioral dynamics: properties

- plan (generate) movement by generating time courses of behavioral variables
- time course of behavioral variables emerge from attractor solutions of a (designed) dynamical system
- that dynamical system is constructed from contributions that express the behavioral constraints

# Behavioral dynamics: target constraint



# Behavioral dynamics: obstacle constraint





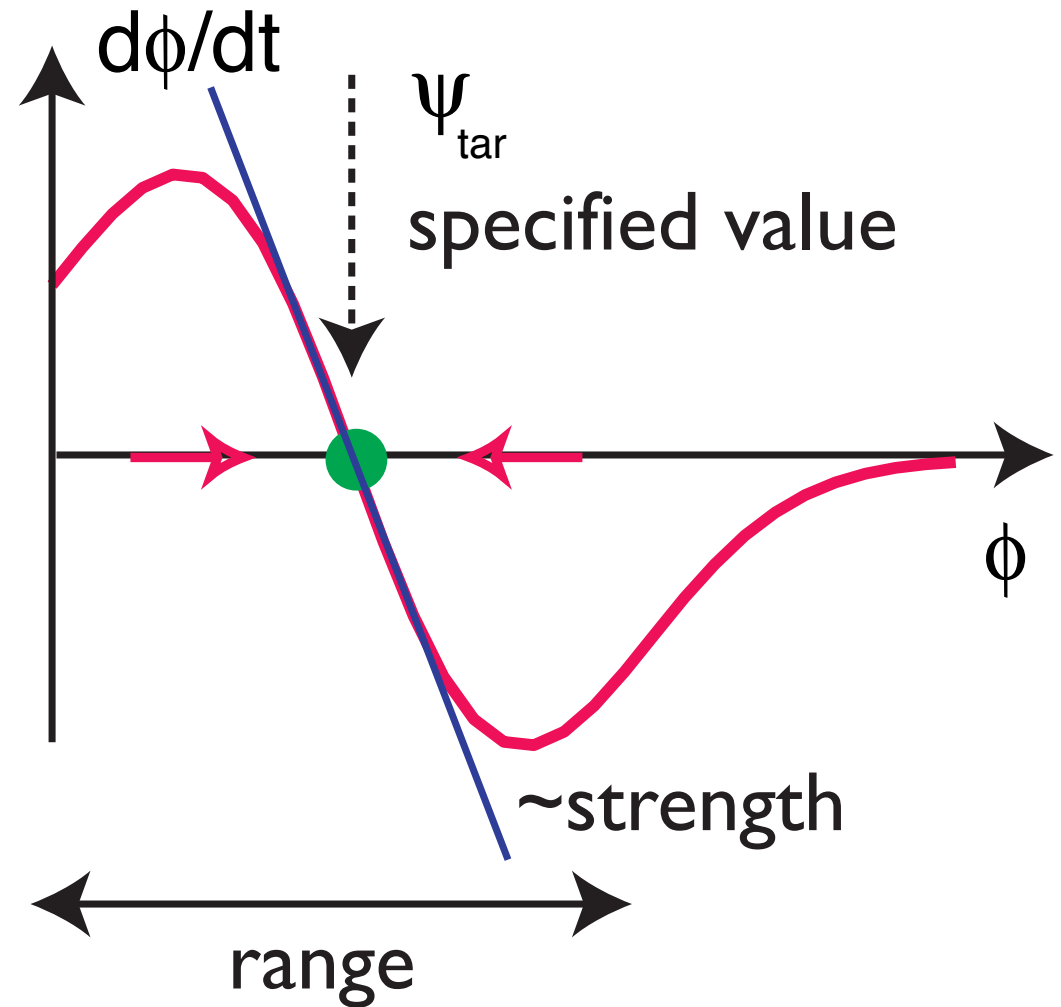
# Behavioral dynamics

■ each contribution is a “force-let” with

■ specified value

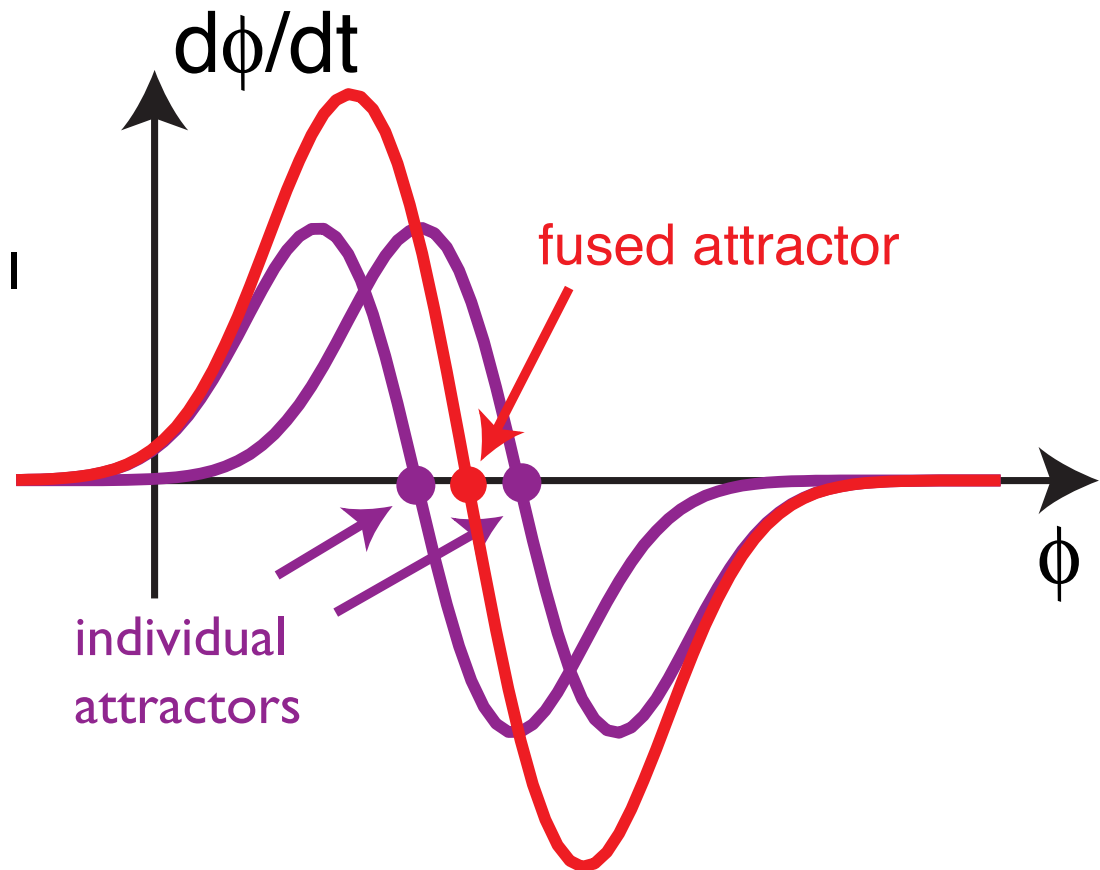
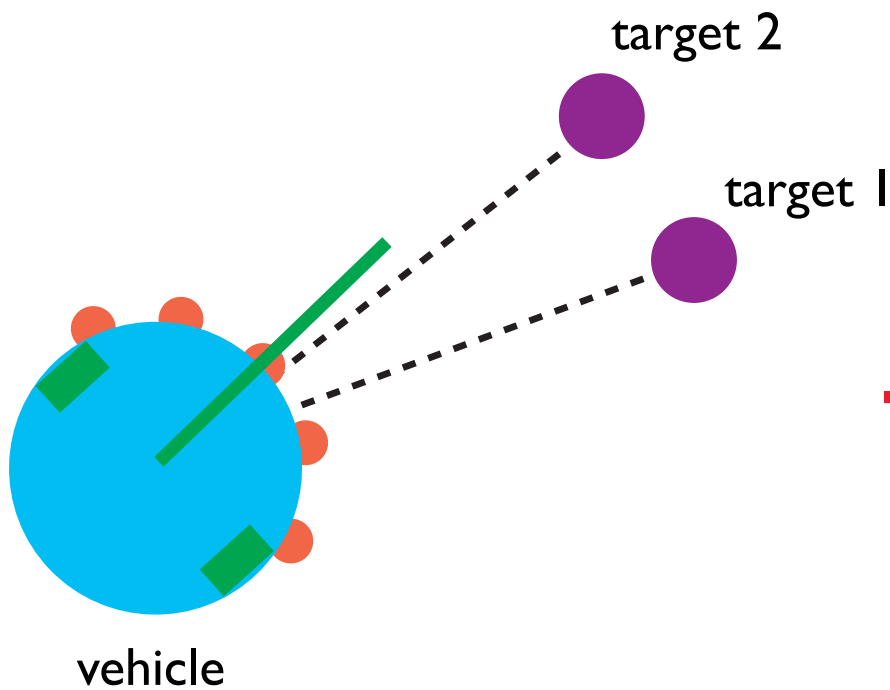
■ strength

■ range



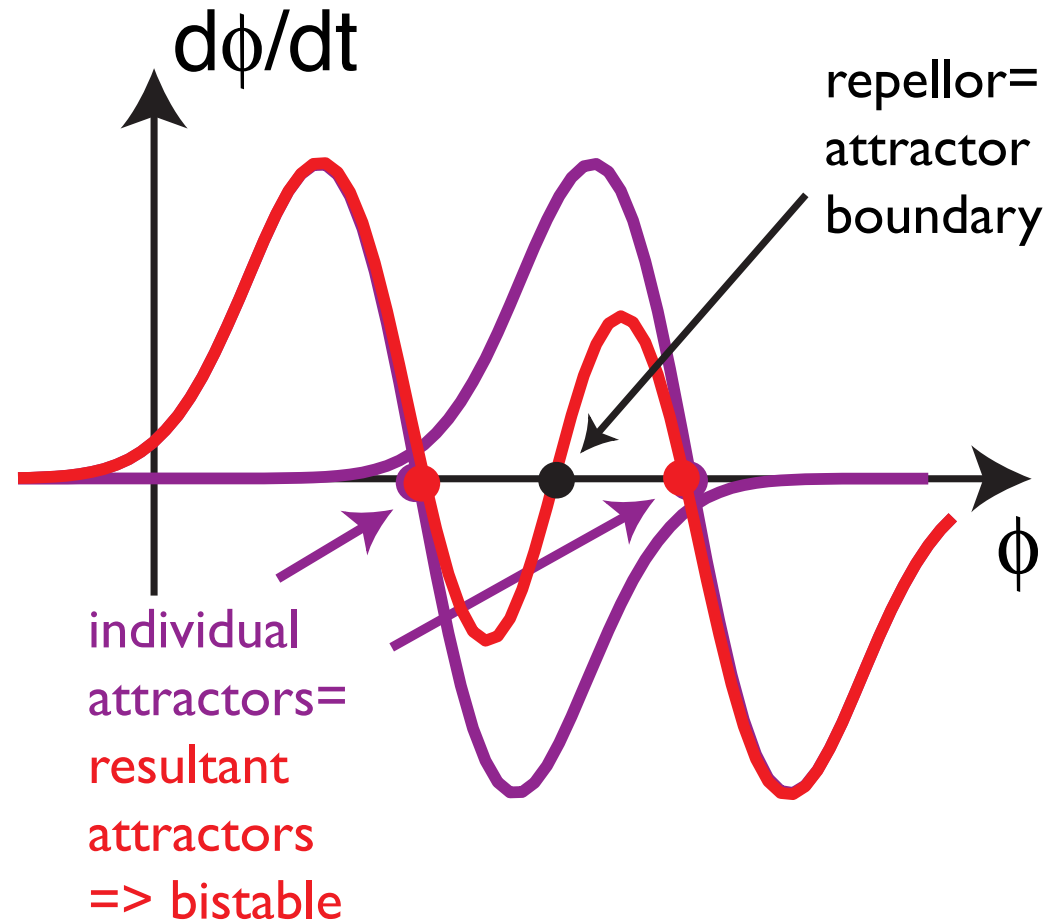
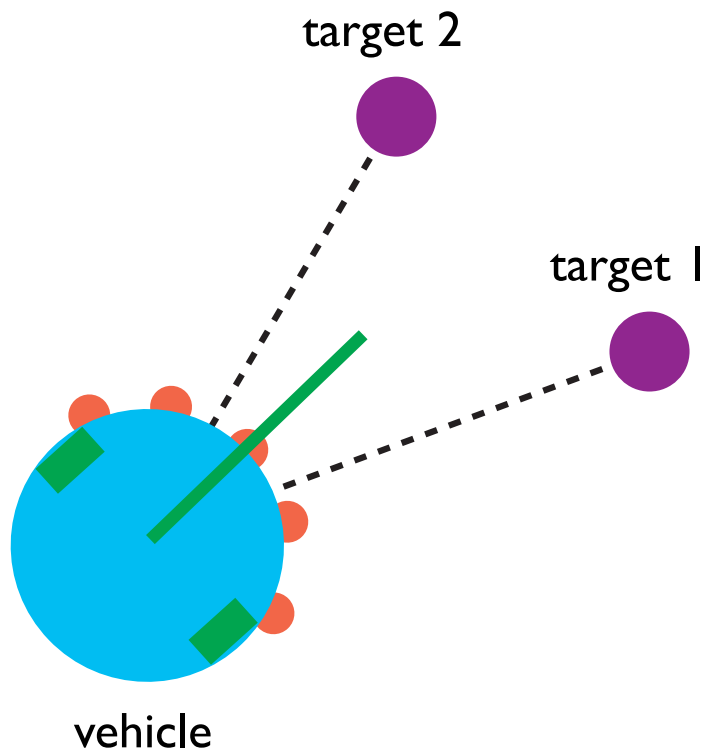
# Behavioral dynamics

- multiple constraints: superpose “force-lets”
- e.g. fuse two target constraints



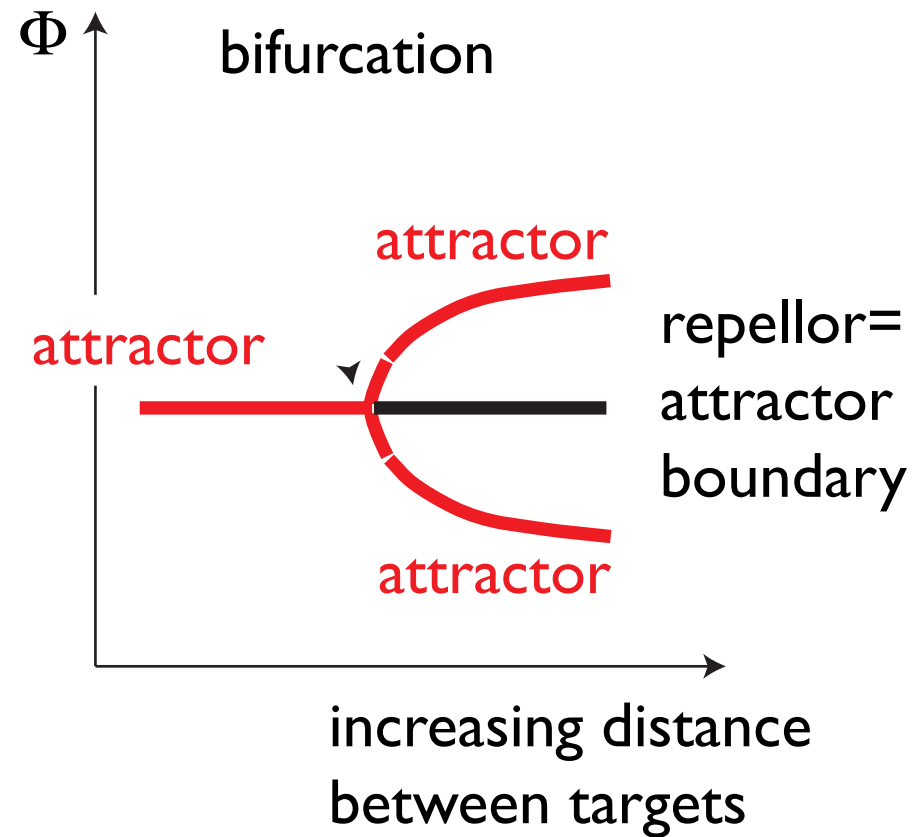
# Behavioral dynamics

- example: select between two targets... decision



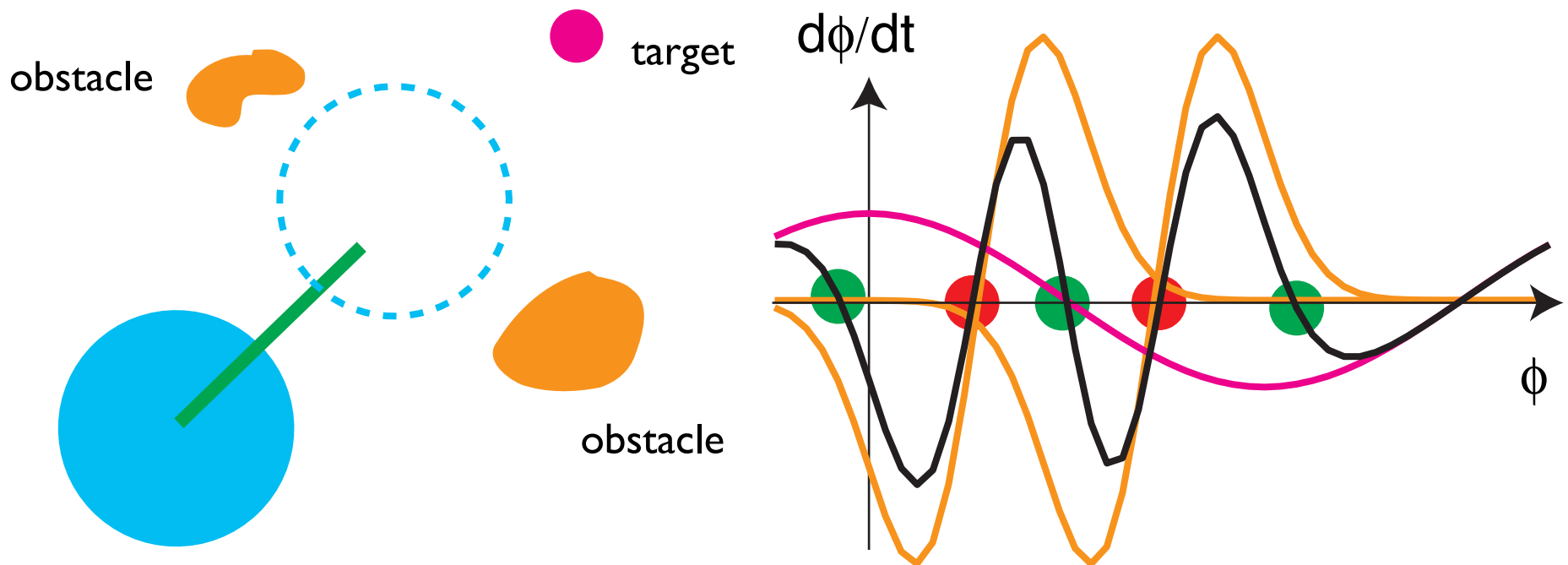
# Behavioral dynamics

- capacity to make decision comes from bifurcation



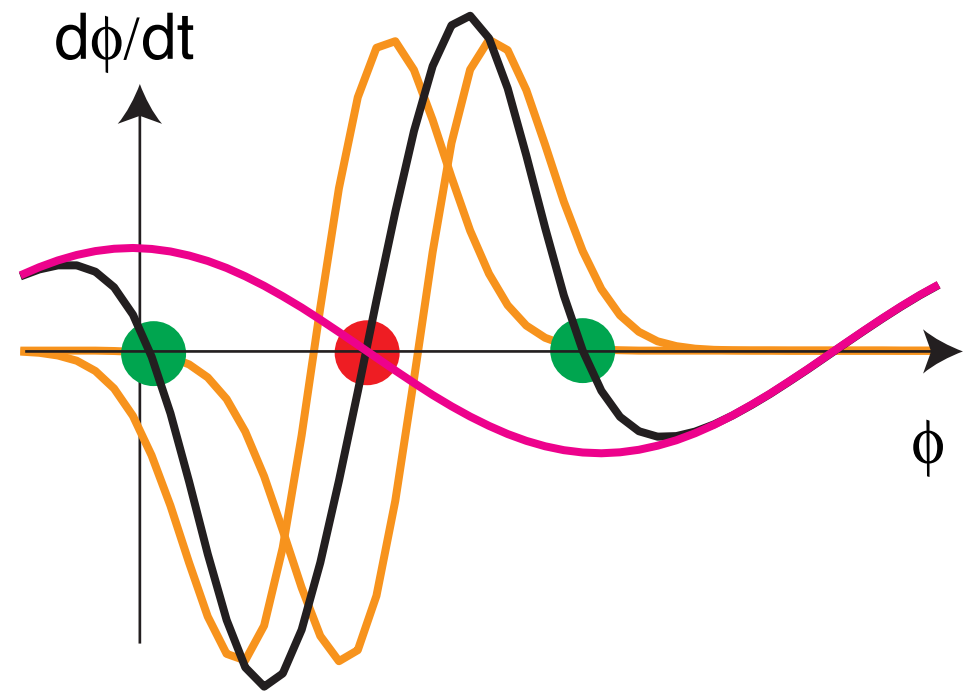
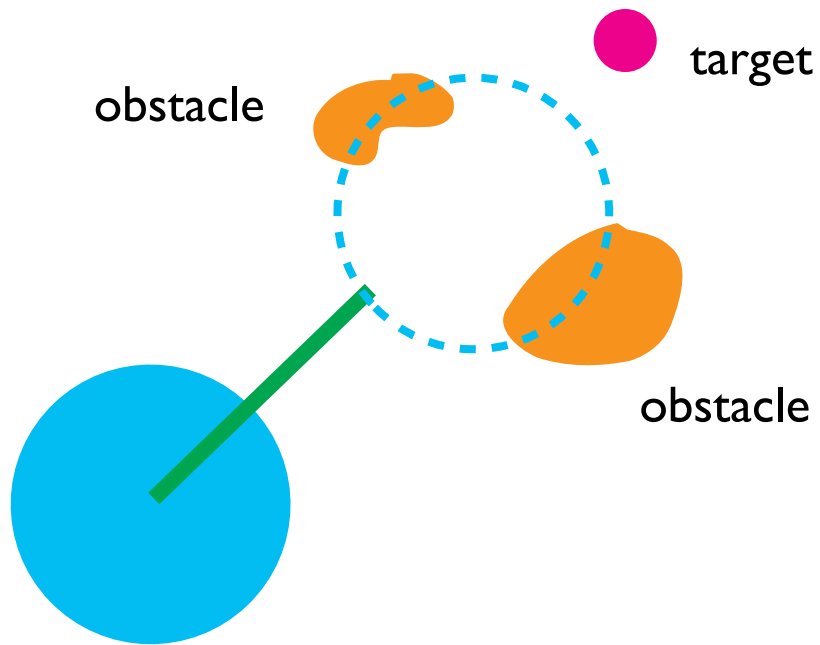
# Behavioral dynamics

- closer to “real life”: bifurcations in obstacle avoidance and target acquisition
- one regime: constraints not in conflict



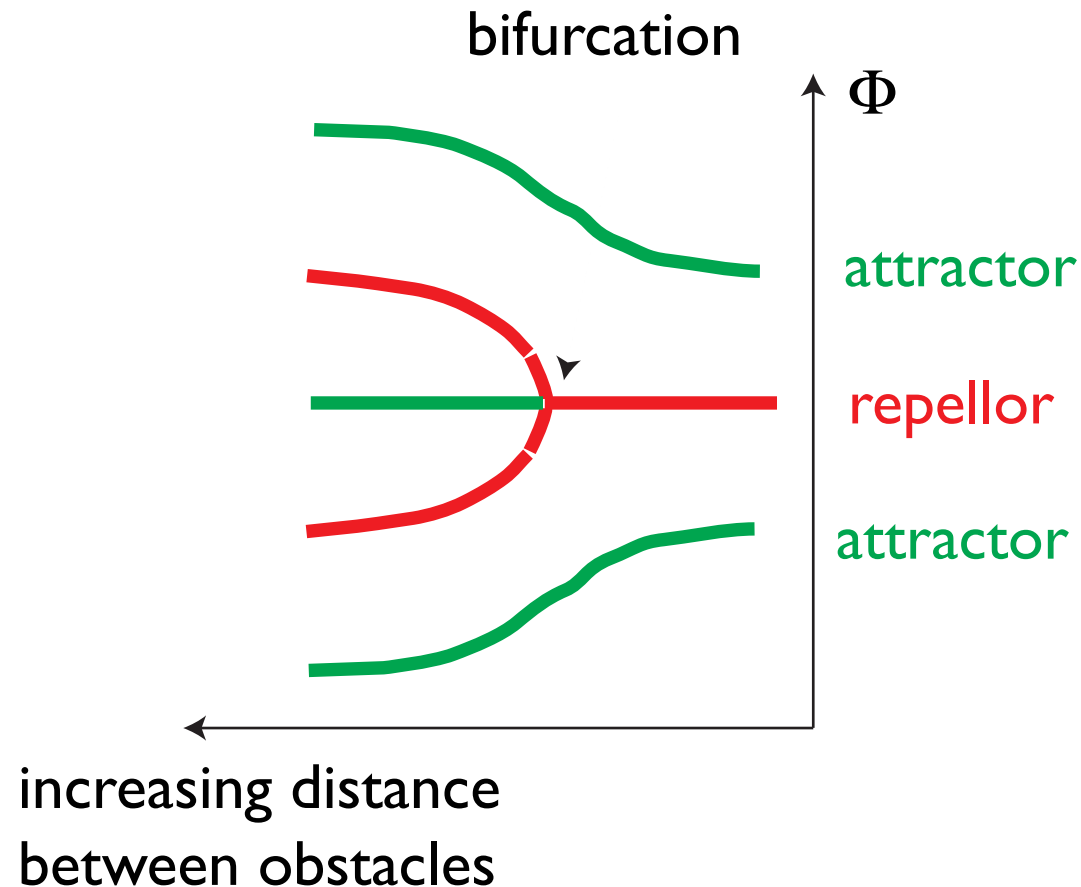
# Behavioral dynamics

- other regime: constraints are in conflict



# Behavioral dynamics

- transition from “constraints not in conflict” to “constraints in conflict” is a bifurcation



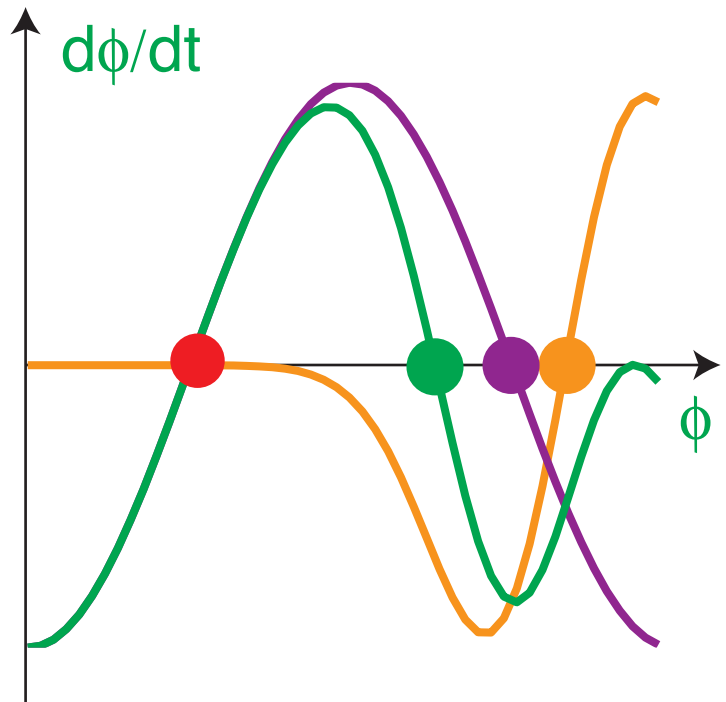
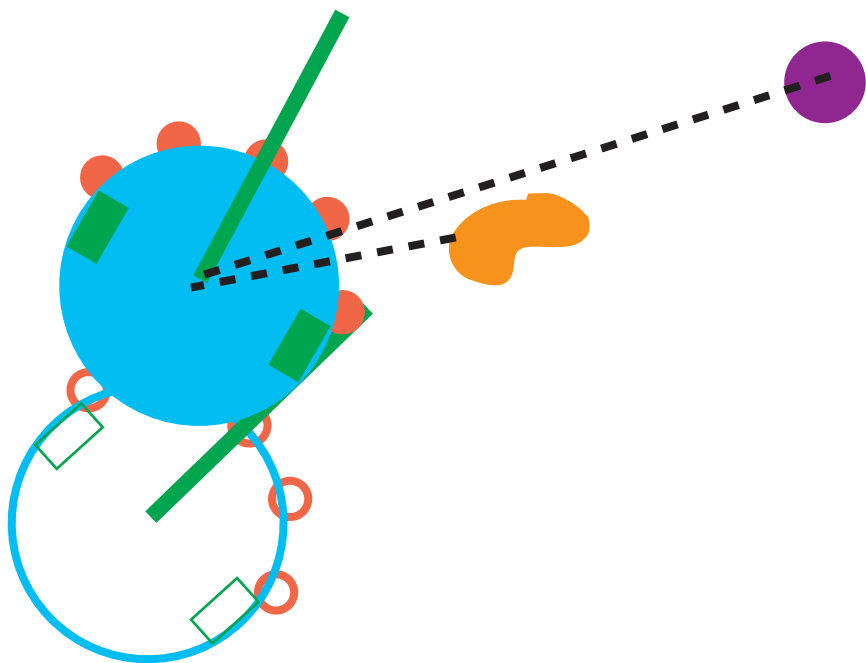
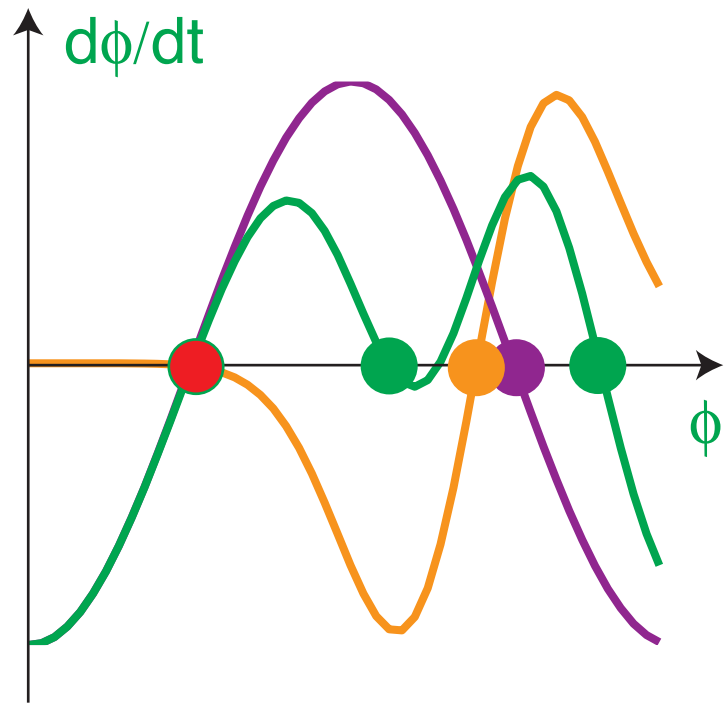
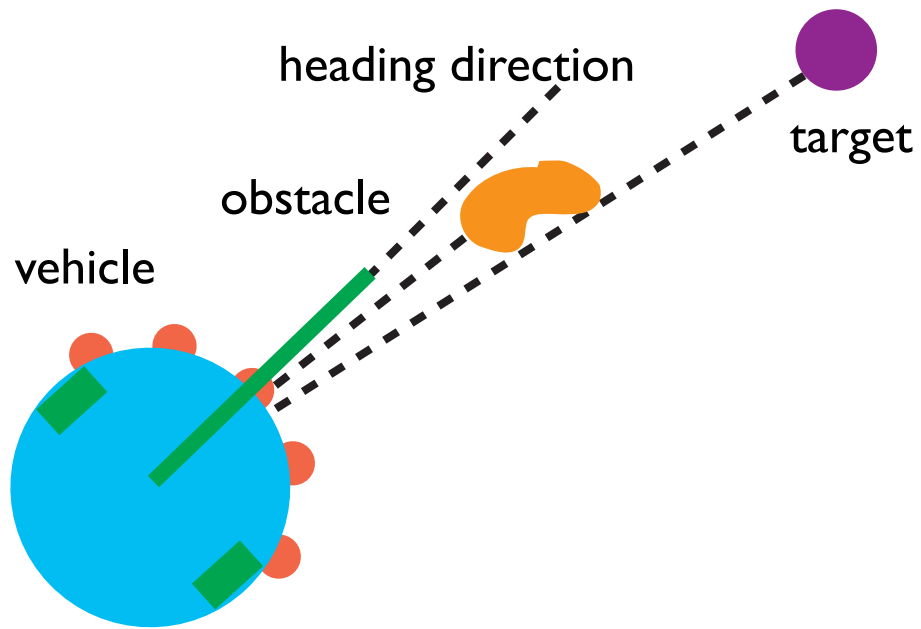
# Behavioral dynamics

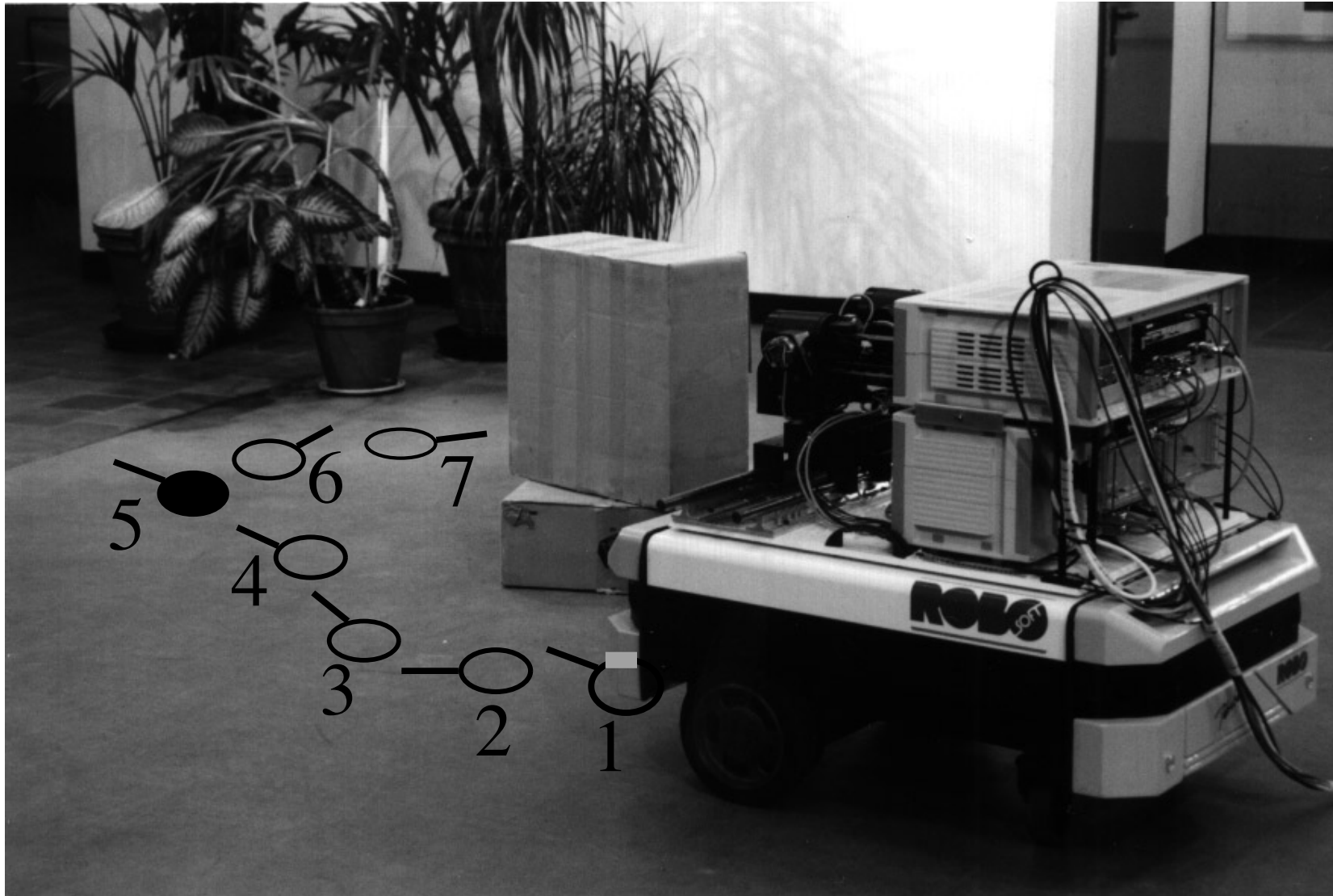
- Such design of decision making is only possible because system “sits” in attractor.
- This reduces the difficult design of the full flow (ensemble of all transient solutions) of non-linear dynamical systems to the easier design of attractors (bifurcation theory).



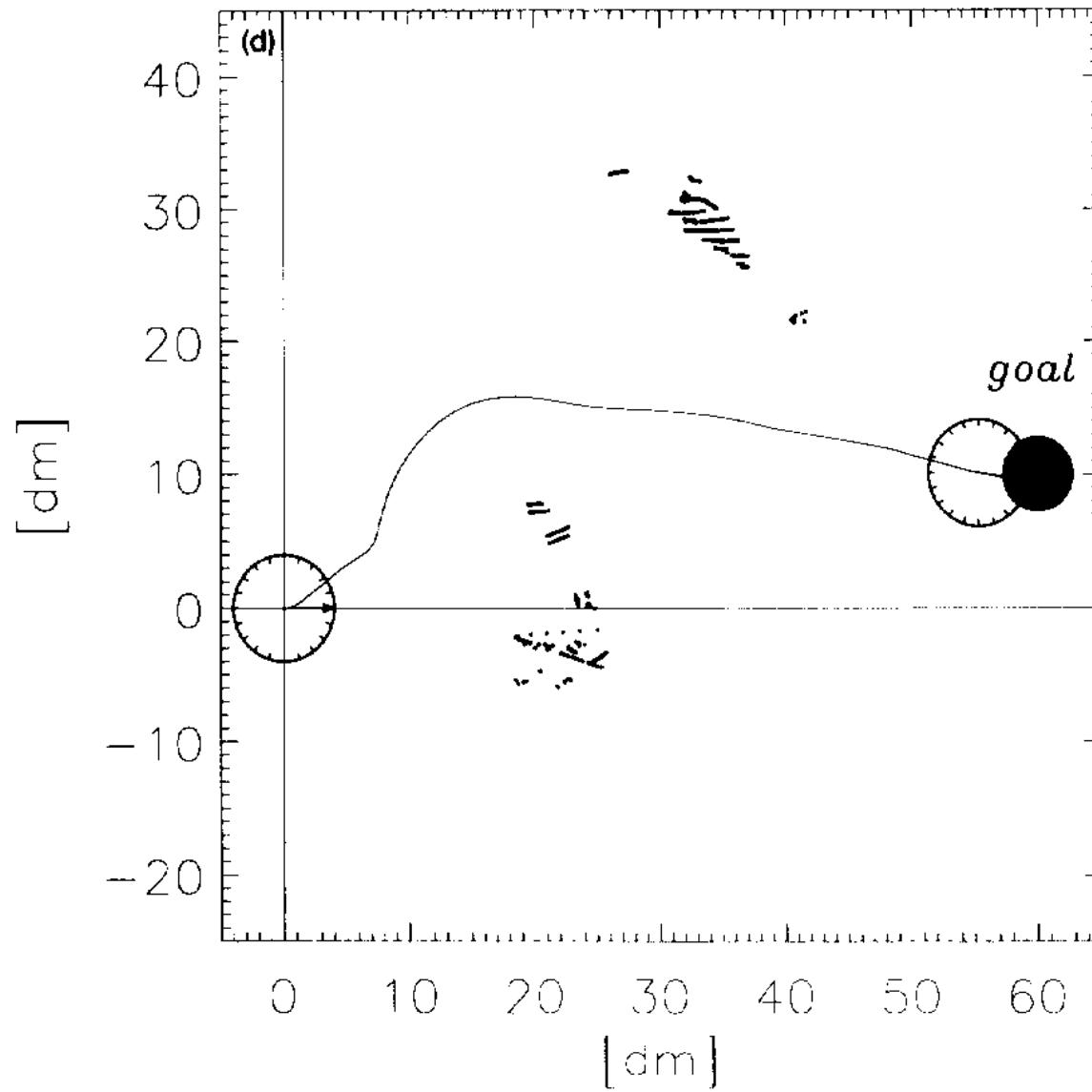
# Behavioral dynamics

- But how may complex behavior be generated while “sitting” in an attractor?
- Answer: force-lets depend on sensory information and sensory information changes as the behavior unfolds





[Schöner, Dose, 1992]



[Schöner, Dose, Engels, 1995]

# The “symbolic” approach

- “obstacles” and “targets” are objects, that have identity, preserved over time...
- implies demands on perceptual systems to deliver such objects and their parameters consistently across time
- next week we’ll look at how a “sub-symbolic” attractor dynamics approach may work directly off low-level sensory information

# Attractor dynamics model of human locomotory movement

- Fajen et al, International Journal of Computer Vision 54(1/2/3), 13–34, 2003 2003

# human locomotion

- Bill Warren and Bret Fajen have used the attractor dynamics approach to account for how humans locomote in virtual reality



# human locomotion to goal

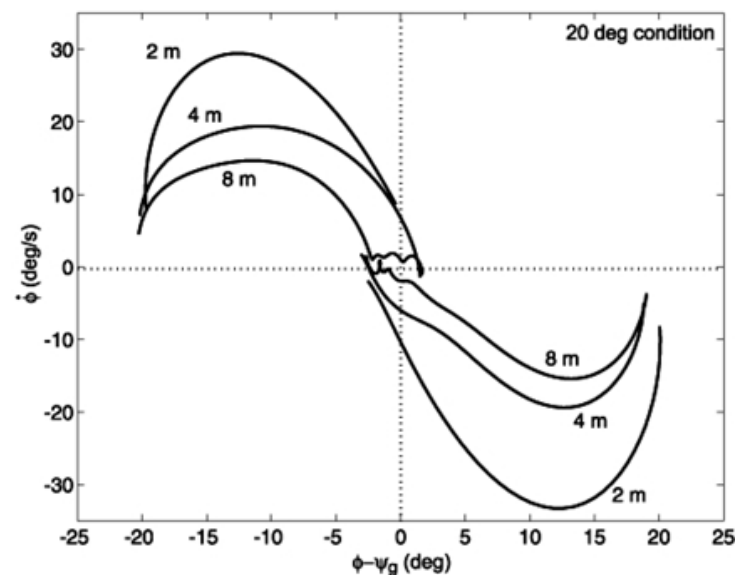
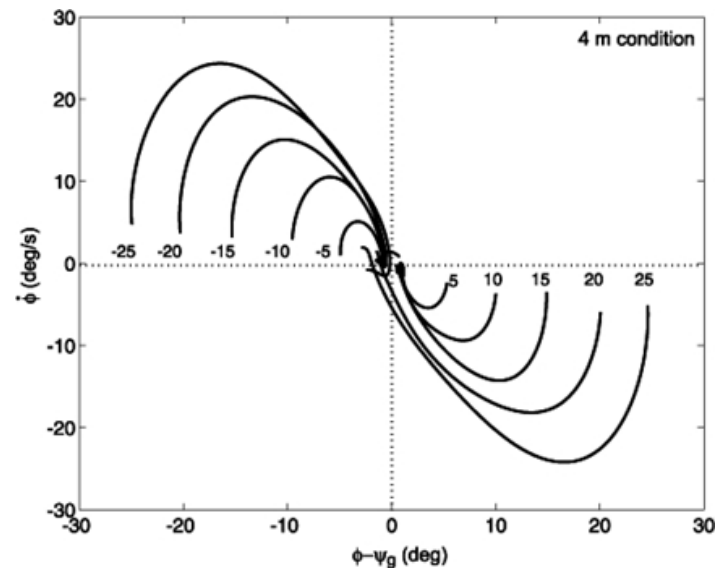
- participants begins to walk
- after walking 1 m, a goal appears at 5, 10, 15, 20, or 25 deg from the straight heading at a distance of 2, 4, or 8 m from participant...
- participants are asked to walk toward the goal



# human locomotion to a goal

■ => turning rate increased with increasing goal angle

■ => turning rate decreased with increasing distance from goal



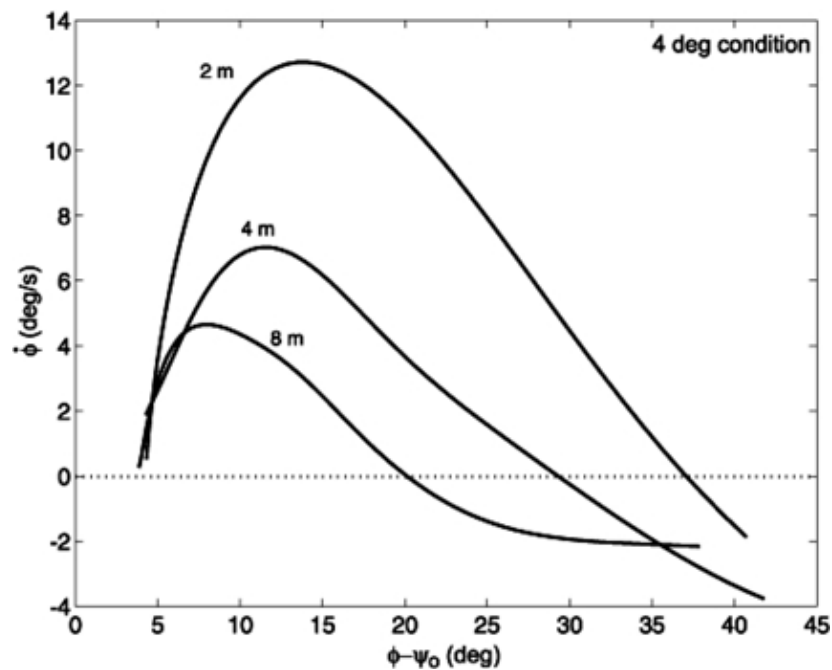
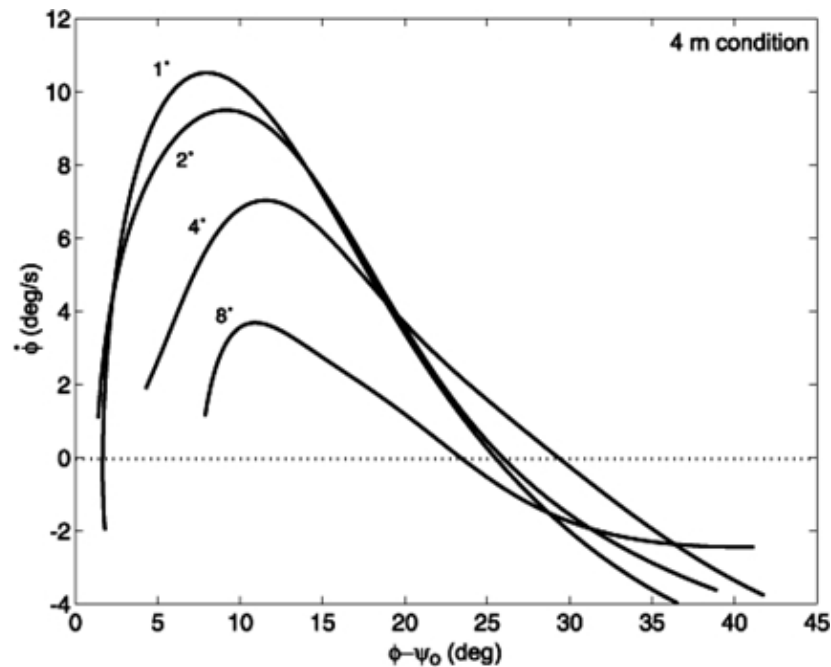
# human locomotion: obstacle

- humans walk toward goal at 10 m distance
- after walking 1 m, an obstacle appears at 1, 2, 4, or 8 deg from heading and a distance of 3, 4, or 5 m

# human locomotion: obstacle

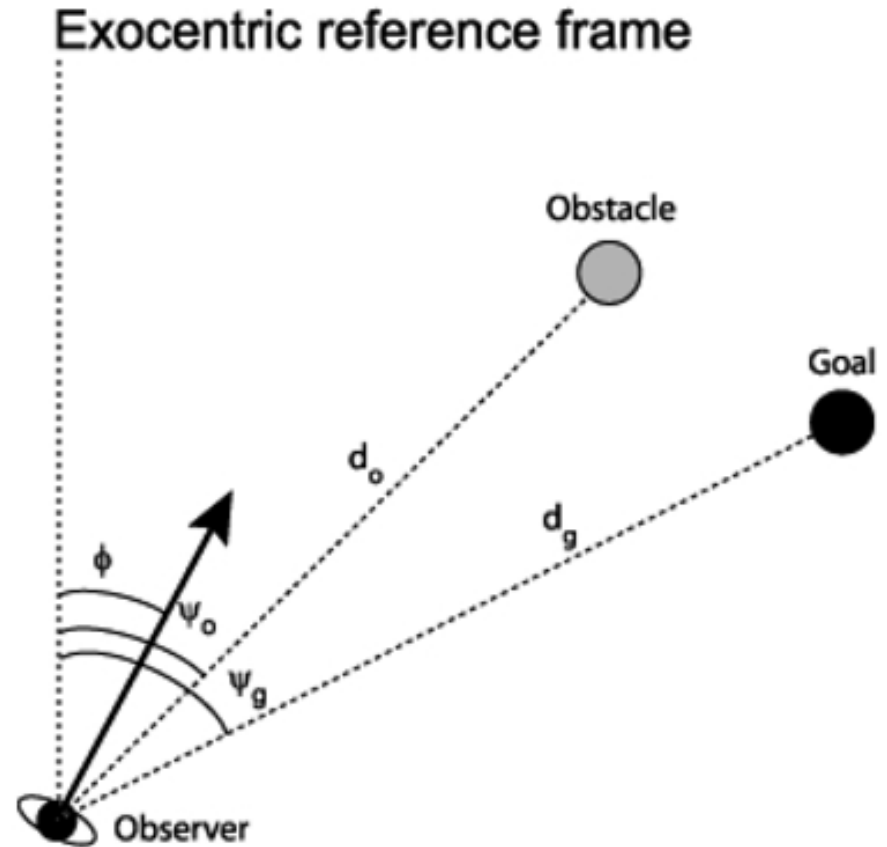
■ => turning rate away from obstacle decreased with obstacle angle

■ => and with obstacle distance



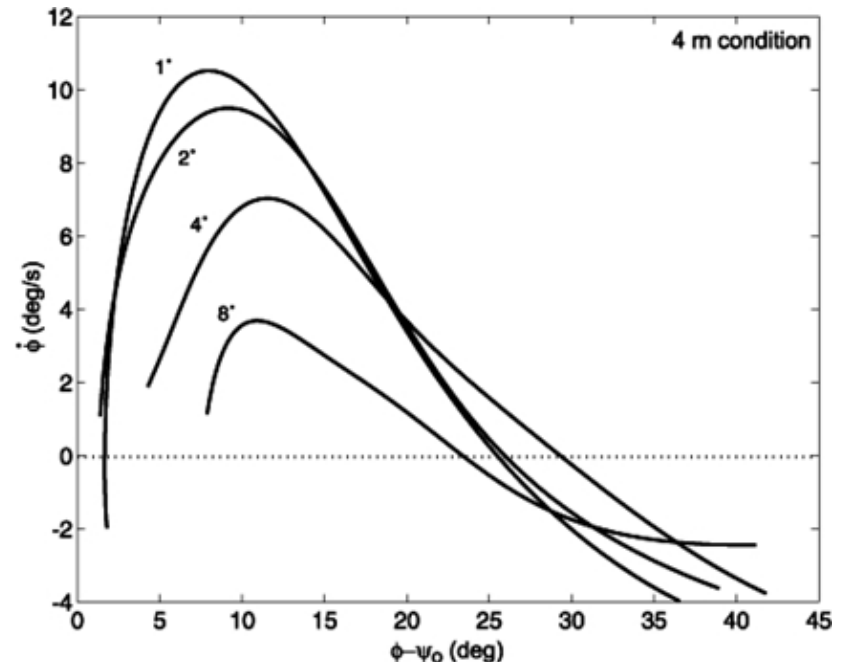
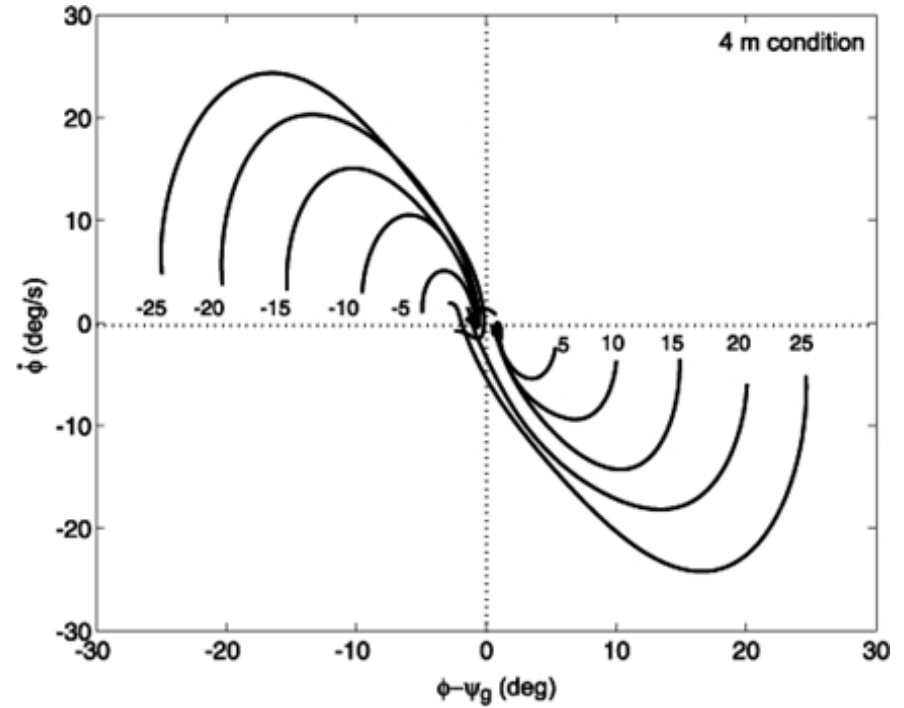
# model

- heading direction as dynamical variable



# model

- first order dynamics  $\dot{\phi} = f(\phi)$  not quite consistent with dependence on initial heading...
- but overall shape of  $\dot{\phi}$  vs  $\phi$  and distance dependence consistent with attractor dynamics approach to heading direction



# attractor dynamics model

- solution: 2nd order dynamics in heading

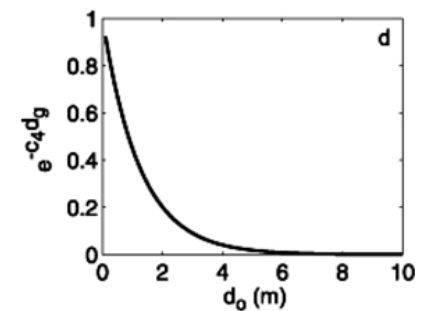
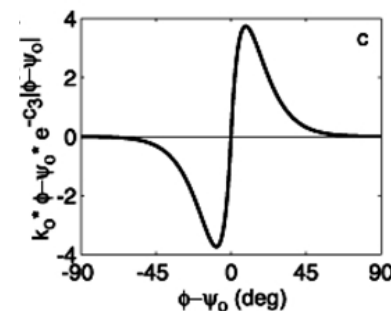
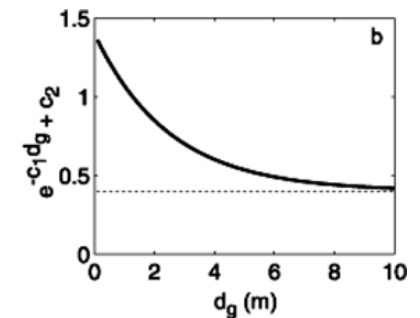
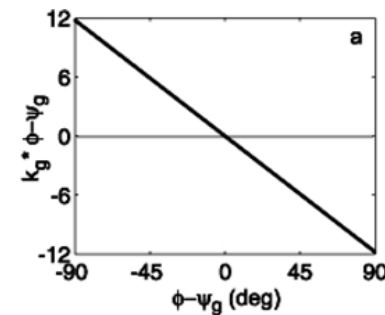
inertial term

damping term

attractor goal heading

$$\ddot{\phi} = -b\dot{\phi} - k_g(\phi - \psi_g)(e^{-c_1 d_g} + c_2) + k_o(\phi - \psi_o)(e^{-c_3|\phi - \psi_o|})(e^{-c_4 d_o})$$

repellor obstacle heading



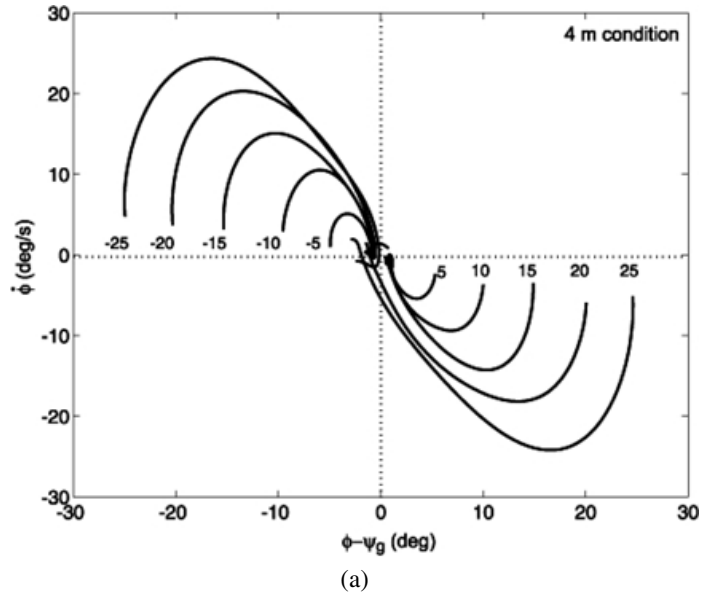
# attractor dynamics model

- approximation: inertia to zero: find first order dynamics with time scale  $b$
- compute fixed points and stability: fixed points of first order dynamics are fixed points too and have the matching stability

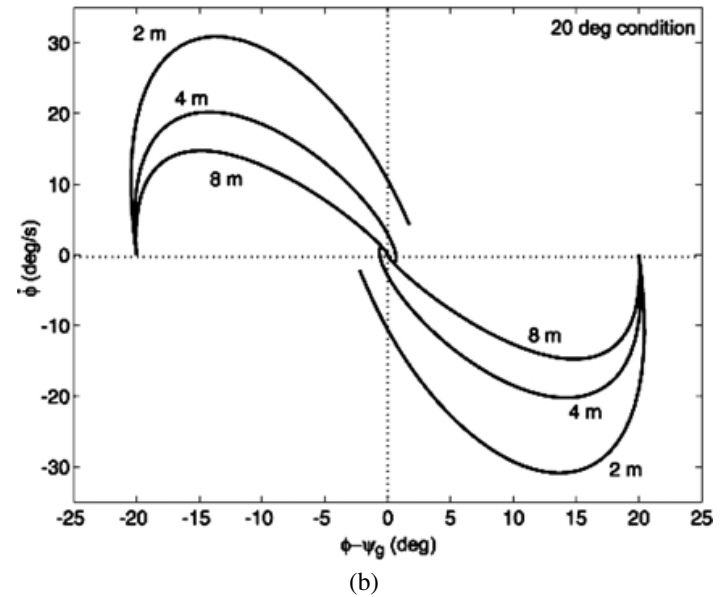
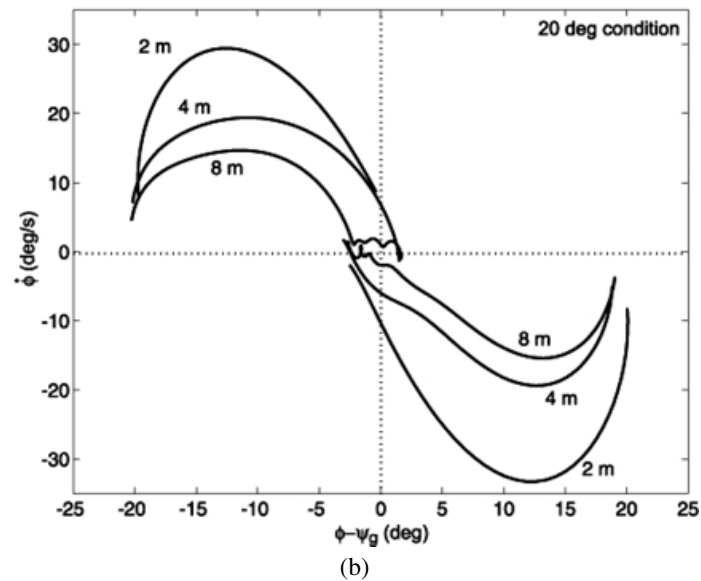
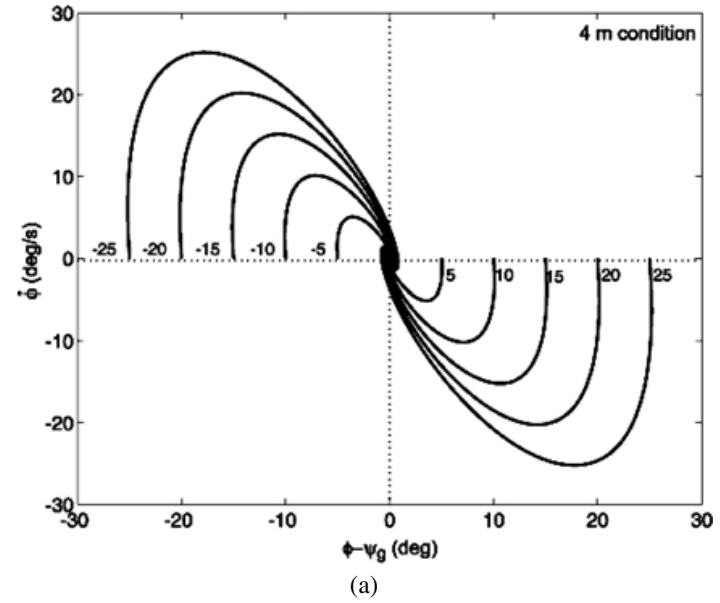
$$\ddot{\phi} = -b\dot{\phi} - k_g(\phi - \psi_g)(e^{-c_1 d_g} + c_2) \quad \text{attractor goal heading} \\ + k_o(\phi - \psi_o)(e^{-c_3|\phi - \psi_o|})(e^{-c_4 d_o}) \quad \text{repellor obstacle heading}$$

# model-experiment match: goal

experiment



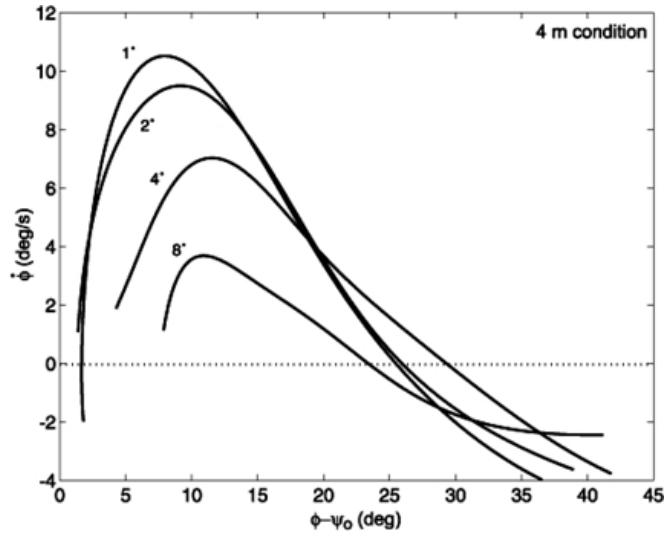
model





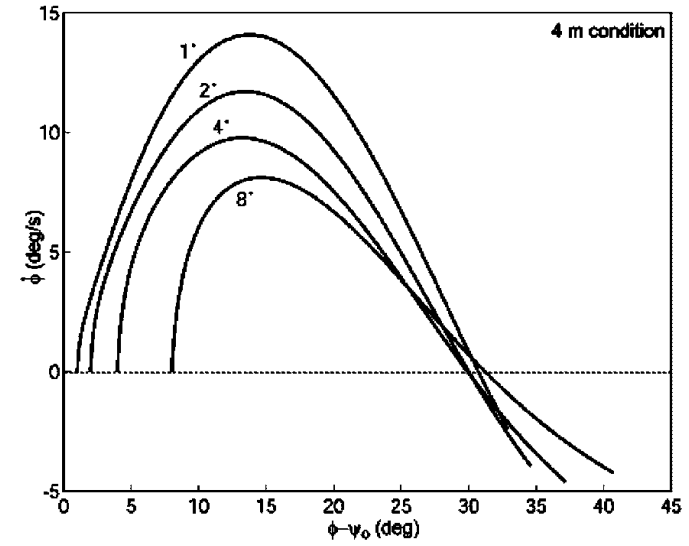
# model-experiment match: obstacle

experiment

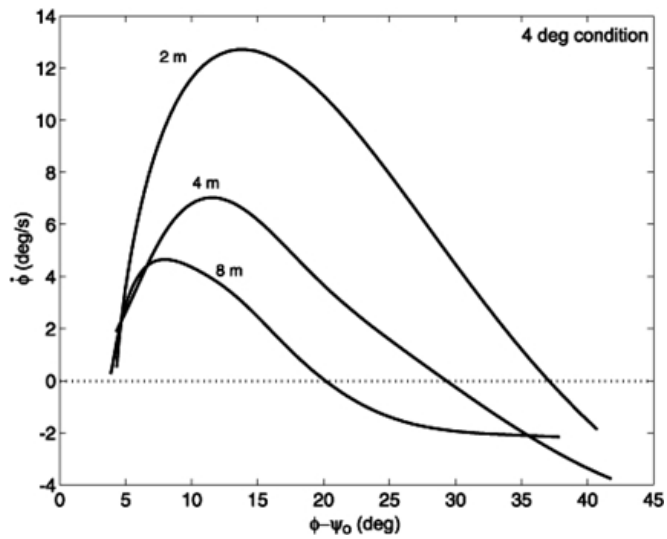


(a)

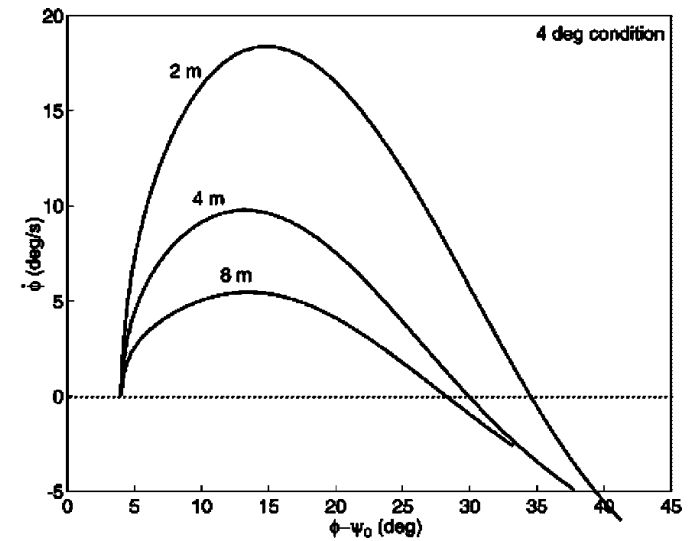
model



(a)

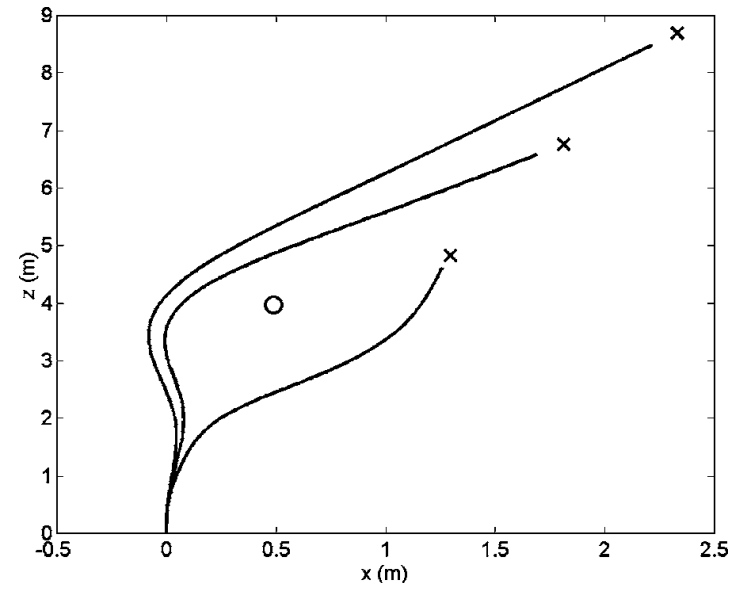
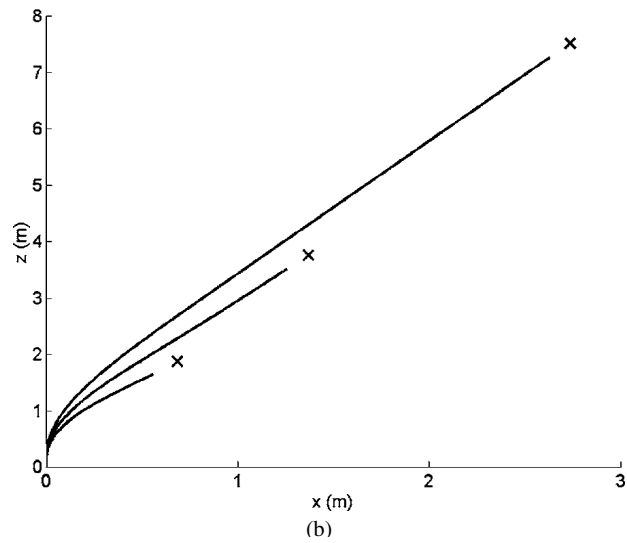
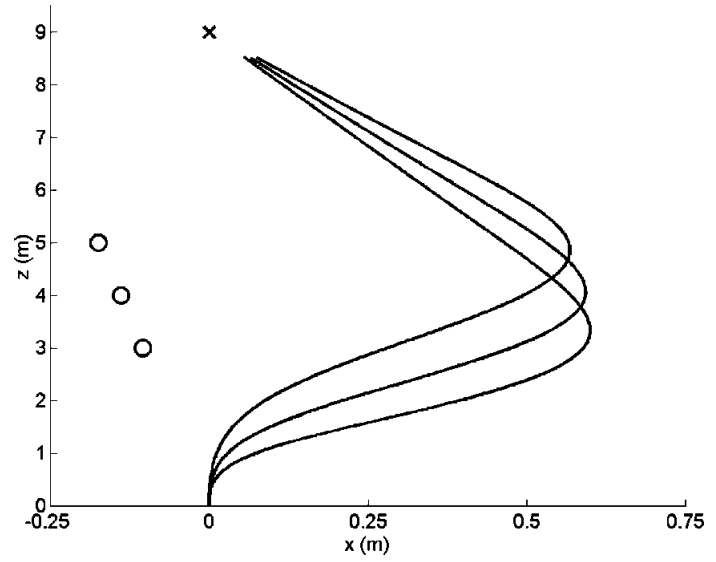
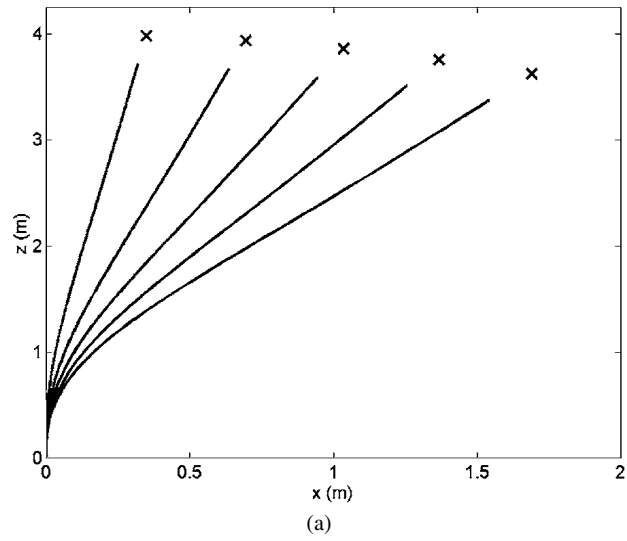


(b)



(b)

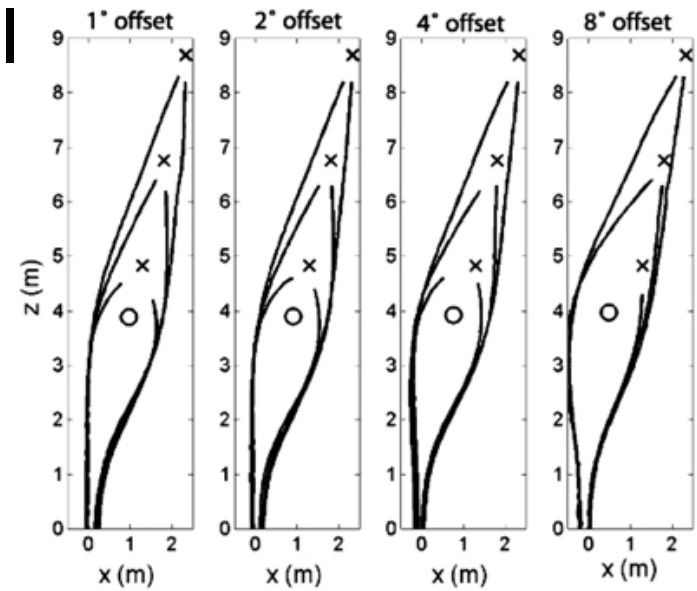
# model: paths



# model-exp: decision making

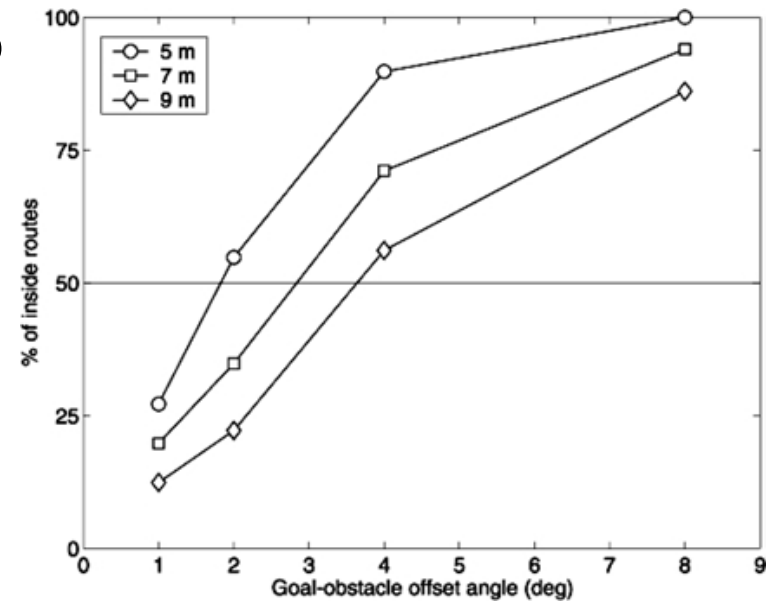
■ inside vs. outside path

model



(a)

exp



(b)

# Basic ideas of the attractor dynamics approach

- plans are time courses of behavioral variables
- these time courses are generated by a dynamical system
- they are structured by attractor solutions of dynamical systems (which may change during the movement)
- decisions emerge from bifurcations of the attractor solutions

# Conclusion

- (symbolic) attractor dynamic approach
  - plans are time courses of behavioral variables
  - generated at attractor solutions of a dynamical system
  - target and obstacle constraints (symbolic) contribute “force-lets’ to the dynamical system
  - decisions emerge from bifurcations of the attractor solutions
- the (symbolic) attractor dynamic account captures for human locomotory behavior in target acquisition and obstacle avoidance