Prof. Dr. G. Schöner, Institute for Neural Computation gregor.schoener@ini.rub.de

Autonomous robotics: Action, Perception, and Cognition

May 3, 2024

Exercise 3 Attractor dynamics for human locomotion

Please upload solutions on the web page before midnight on **May 16, 2024** (Thursday).

Read the paper by Fajen, Warren, Temizer, Kaebling: "A Dynamical Model of Visually-Guided Steering, Obstacle Avoidance, and Route Selection" (*International Journal of Computer Vision* **54**:1334 (2003)) available on the course webpage, to get an overall sense of what their project is about. No need to dig into all details. Then focus on the part relevant to the questions below.

The model of Fajen et al. is described by their Equation (4). Take only the attraction term into account:

$$\ddot{\phi} = -b\dot{\phi} - k(\phi - \psi_g)$$

where the various constants have been contracted into $k > 0$ and $b > 0$. You can further simplify this by introducing a shifted variable $\theta = \phi - \psi_g$.

1. Compute the fixed point. [Hint: Transform the second order equation into a first order equation by introducing an auxiliarly variable $\omega = \dot{\phi}$]

2. Write this linear dynamics as

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \theta \\ \omega \end{pmatrix}$$

where **A** is a matrix. Compute the eigenvalues of that matrix. [Hint: If you don't know how to do this, there is an example in Scheinermann's book, e.g., on page 62, after Eq. 2.10 there. There is also an appendix that states this in general on page 266, A.1.3., after Eq. A.2. Scheinerman's book can be obtained here: `https://github.com/scheinerman/InvitationToDynamicalSystems`]

3. Determine the stability of the fixed point by determining if the Eigenvalues have negative real parts.