

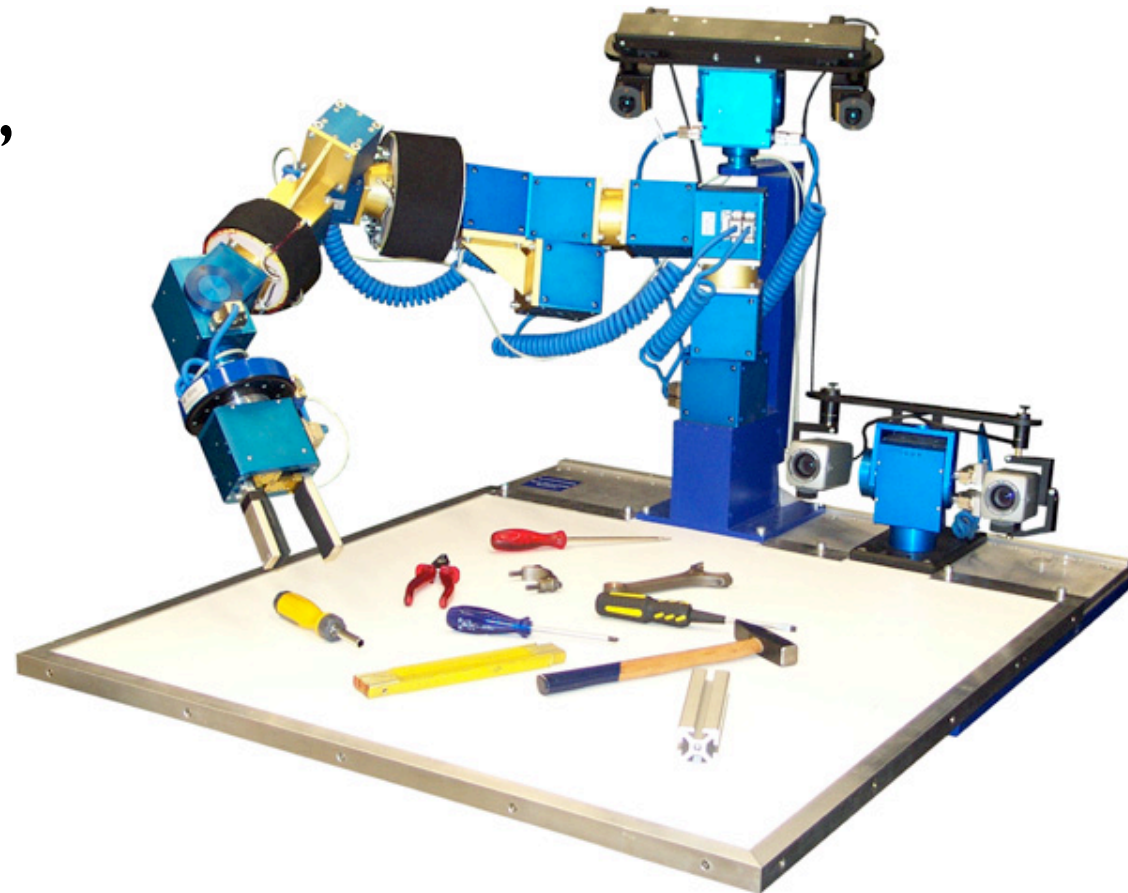
# Robotic manipulation: overview and basic concepts

Gregor Schöner

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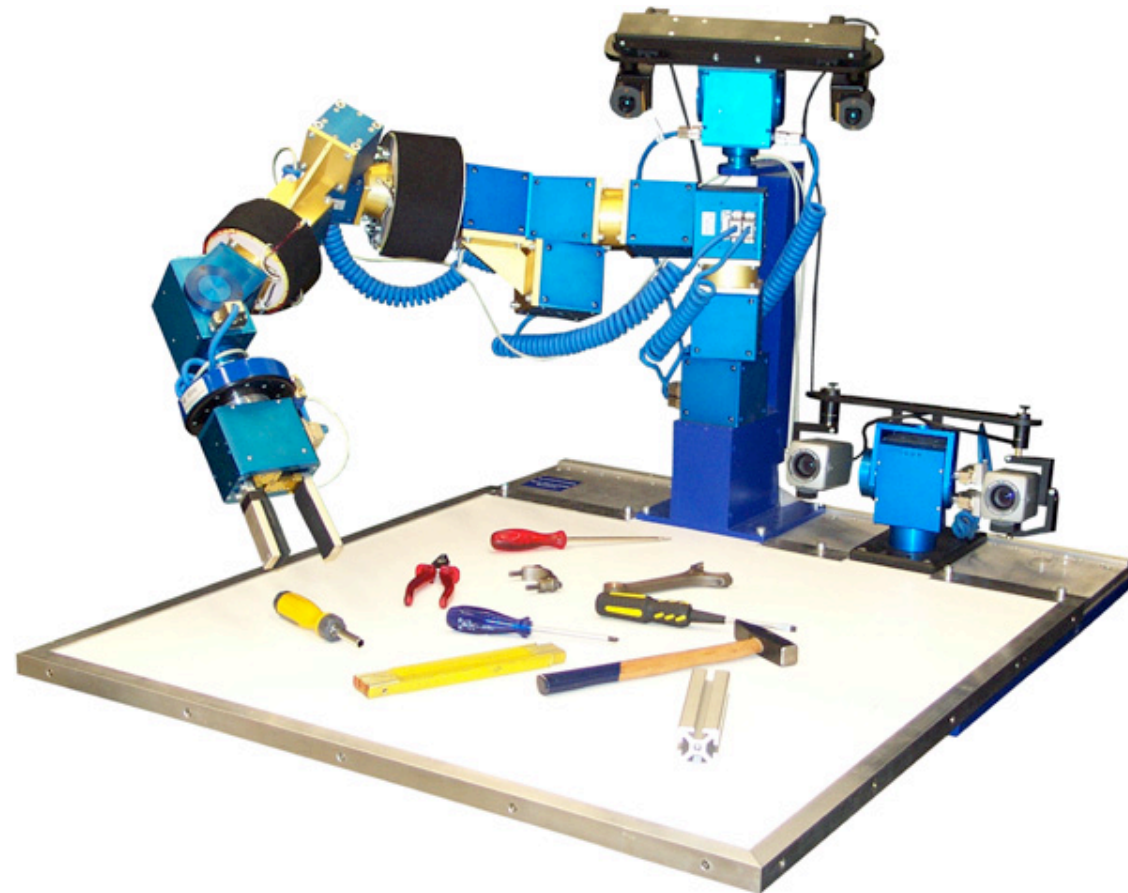
# Movement to reach and grasp, lift, transport, manipulate

- involves a “manipulator”, a robotic/human arm with a grasping mechanism/hand



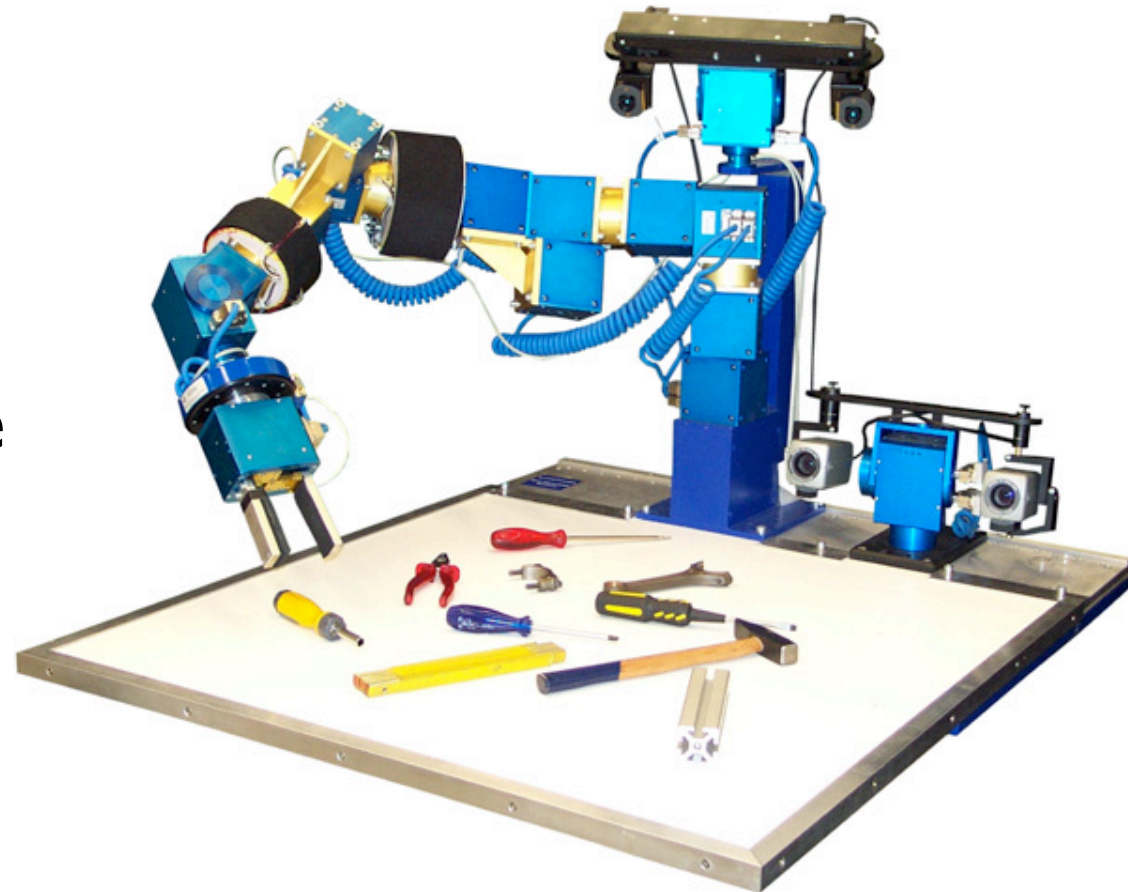
# What is entailed in autonomously reaching for objects?

- Perception: recognizing and segmenting objects, estimating their pose
- Scene representation: registering the spatial array in the arms workspace for possible target objects, free space, and obstacles



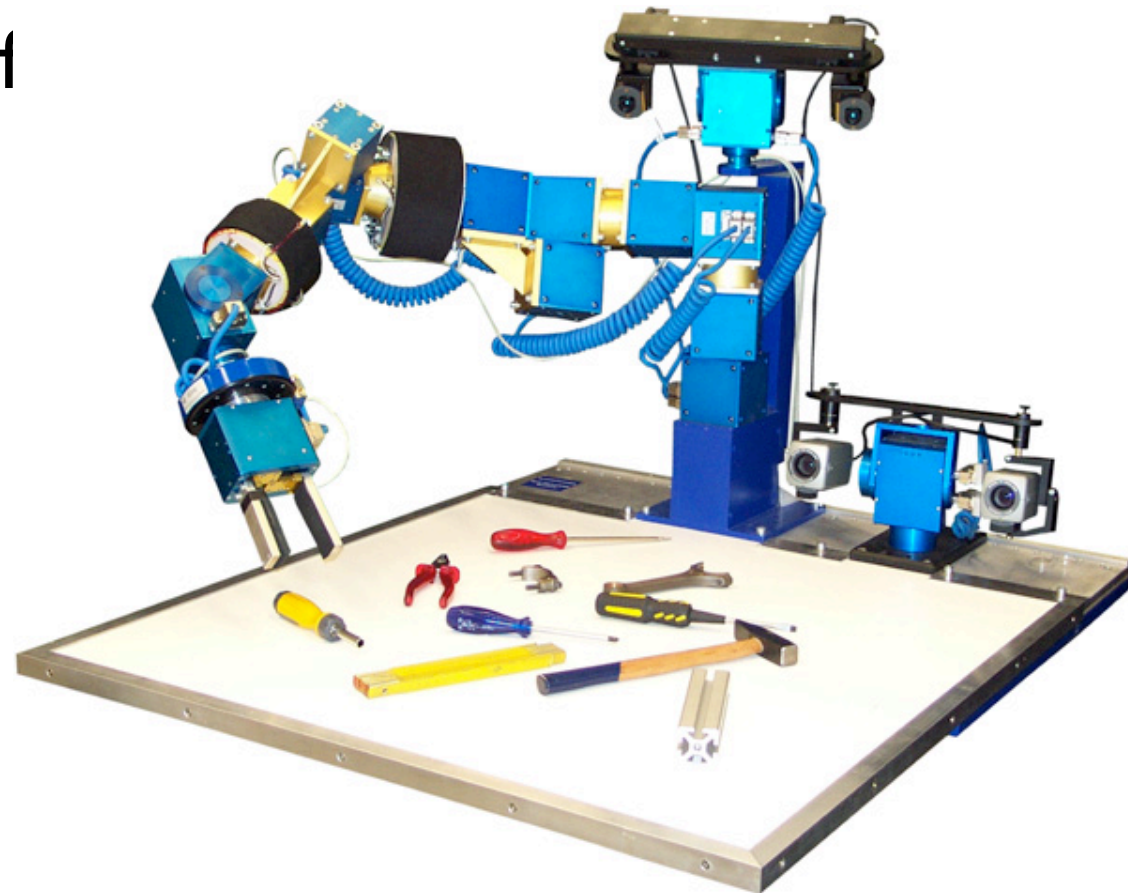
# What is entailed in autonomously reaching for objects?

- Sequentially organizing actions (“serial order”) and planning
- Selecting a relevant object or location in the scene



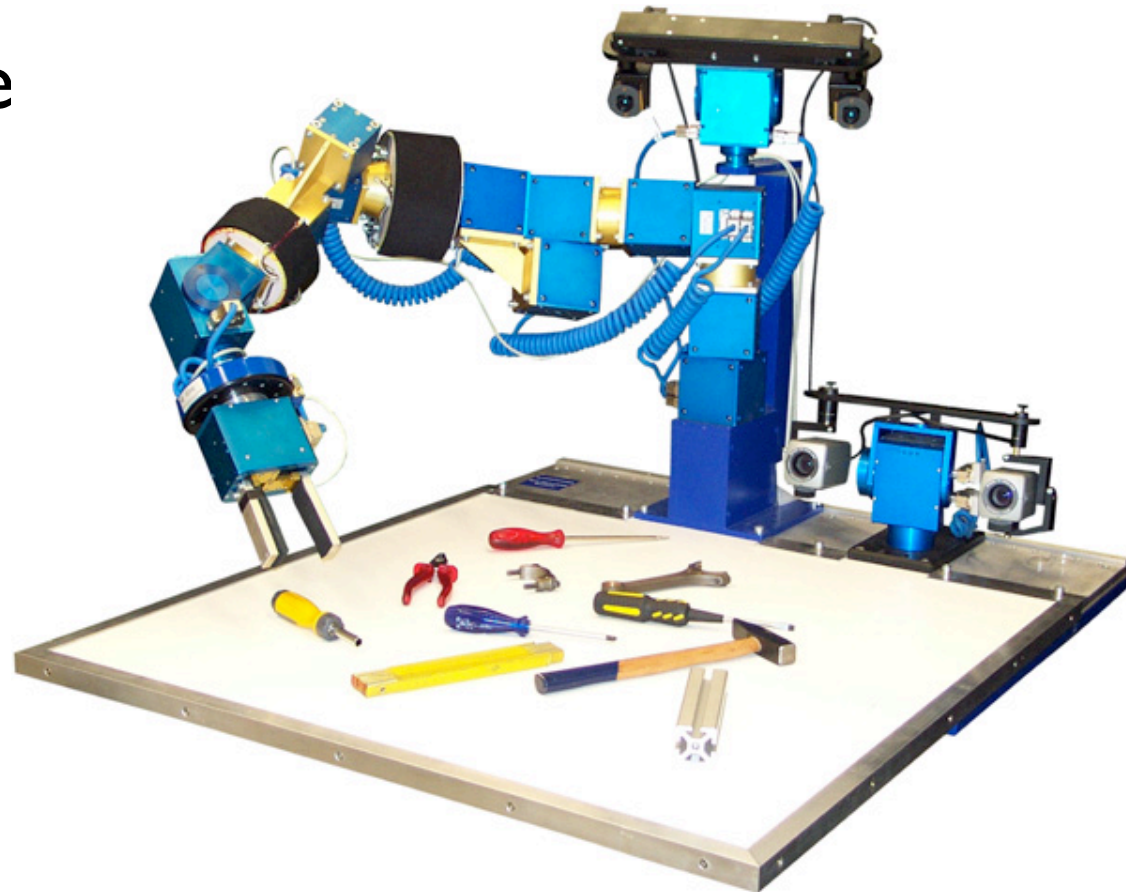
# What is entailed in autonomously reaching for objects?

- Extracting parameters of an individual movement segment based on initial posture of arm and target state



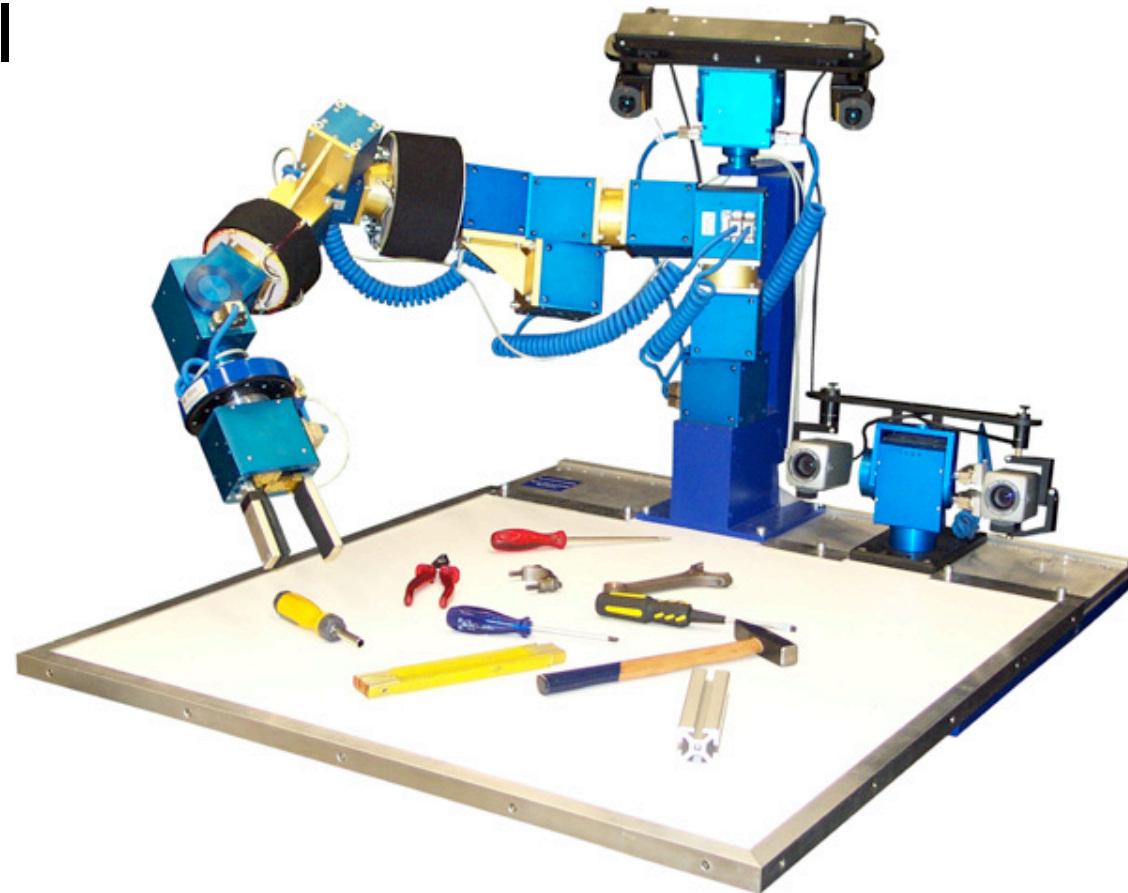
# What is entailed in autonomously reaching for objects?

- Generating a time course for the degrees of freedom of the arm and hand that moves the arm from its initial posture to a state in which the target object is grasped
- Coordinating timed movement



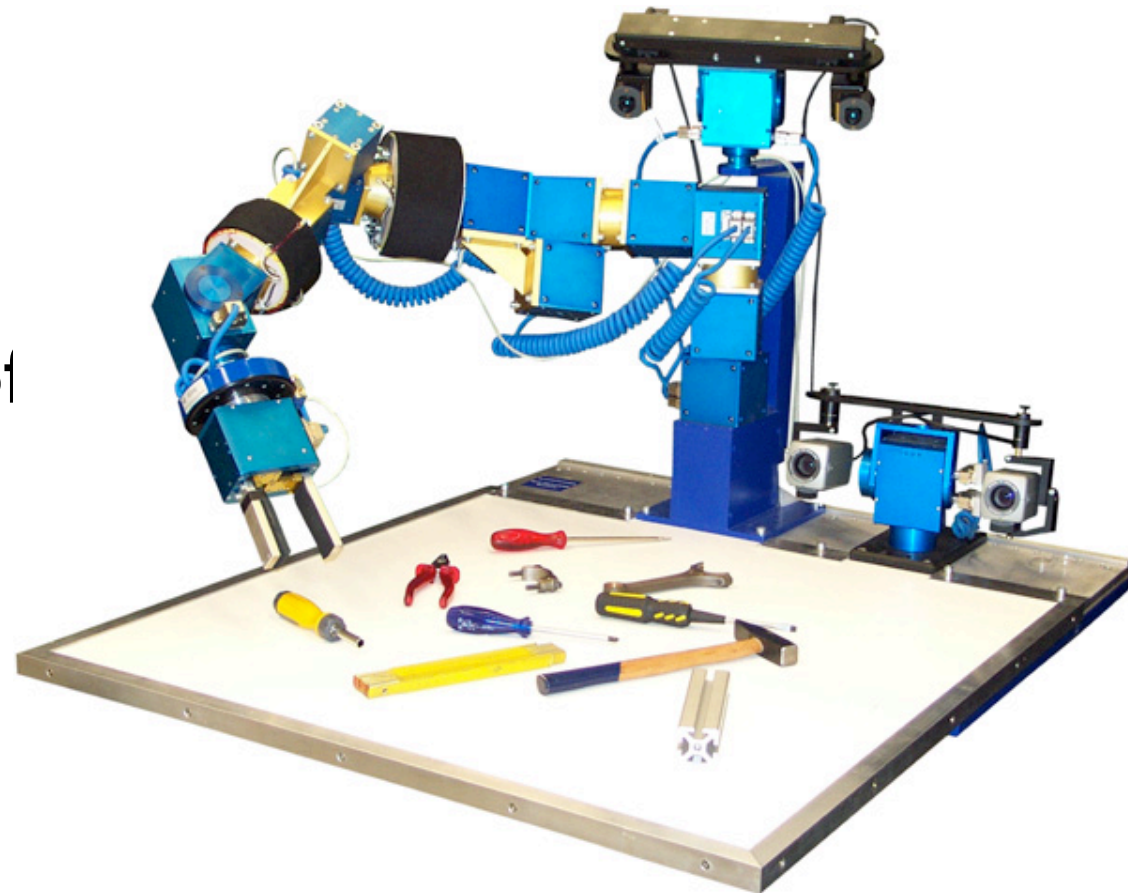
# What is entailed in autonomously reaching for objects?

- Controlling the arm: translating the desired time course into control signals to the actuators/muscles that move the arm
- potentially update these signals based on feedback



# What is entailed in autonomously reaching for objects?

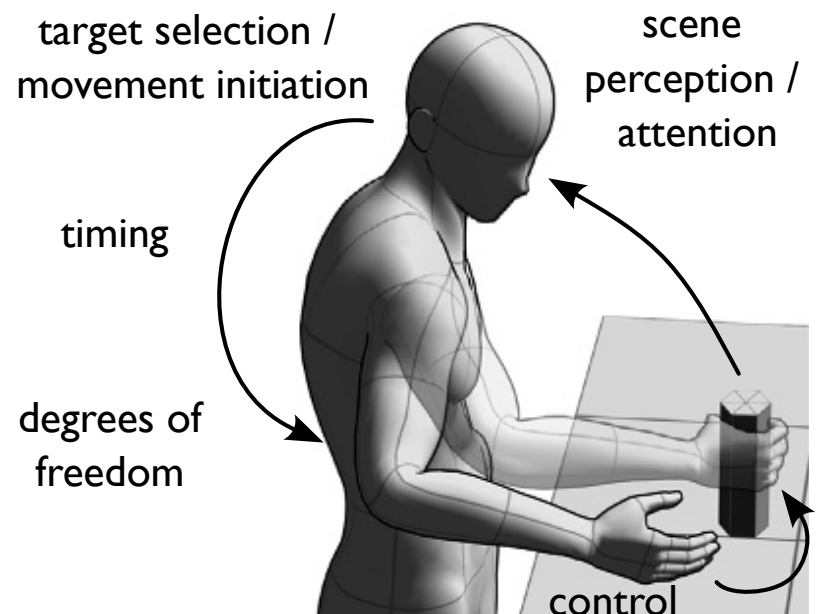
- Detect termination of the movement
- Transition to the next element in a sequence of movements...





# Object-directed action by humans/neural processes

- scene perception, object perception
- movement planning
- movement organization
- trajectory formation
- control



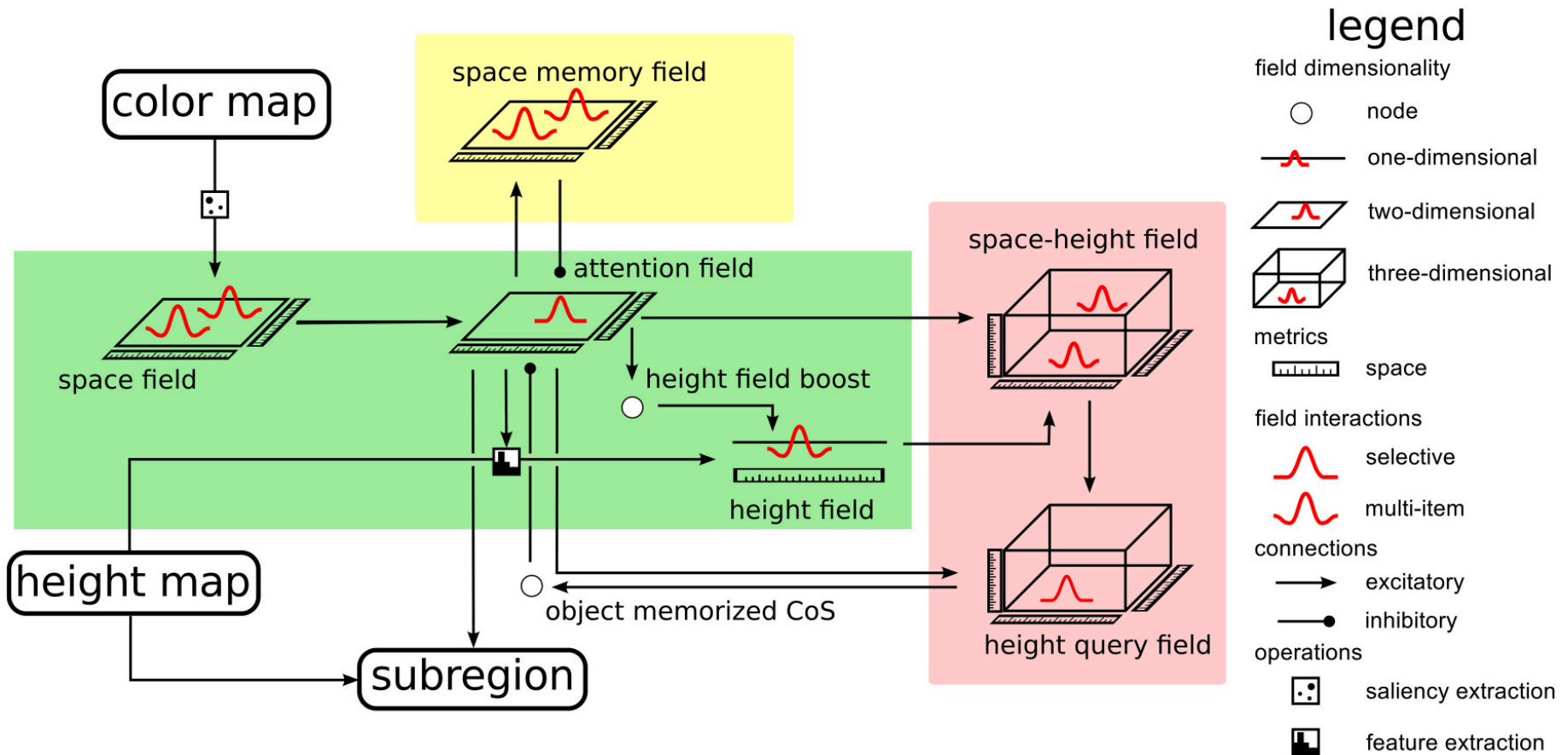
# Perception

- attention, attentional selection
- recognition/classification
- estimation
- segmentation



# Perception

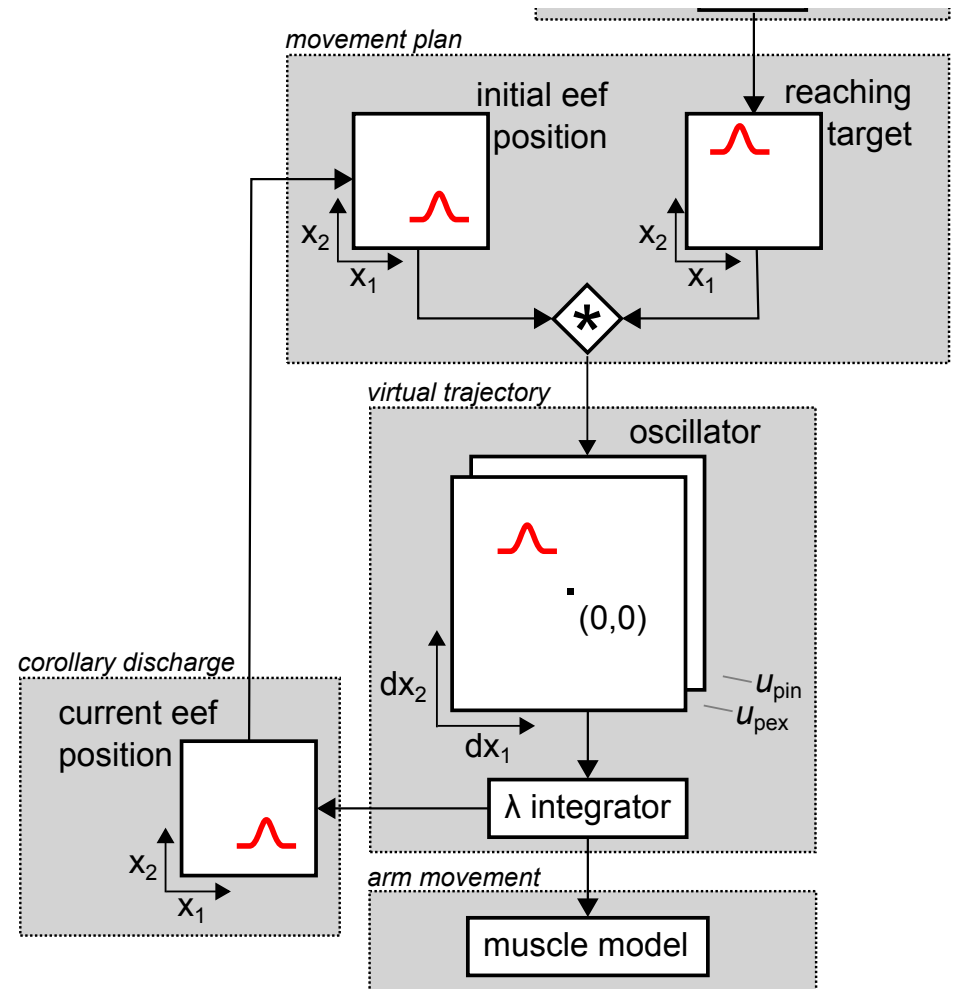
■ attentional selection => pose



[Knips et al, Frontiers Neurobotics 2017]

# Movement planning

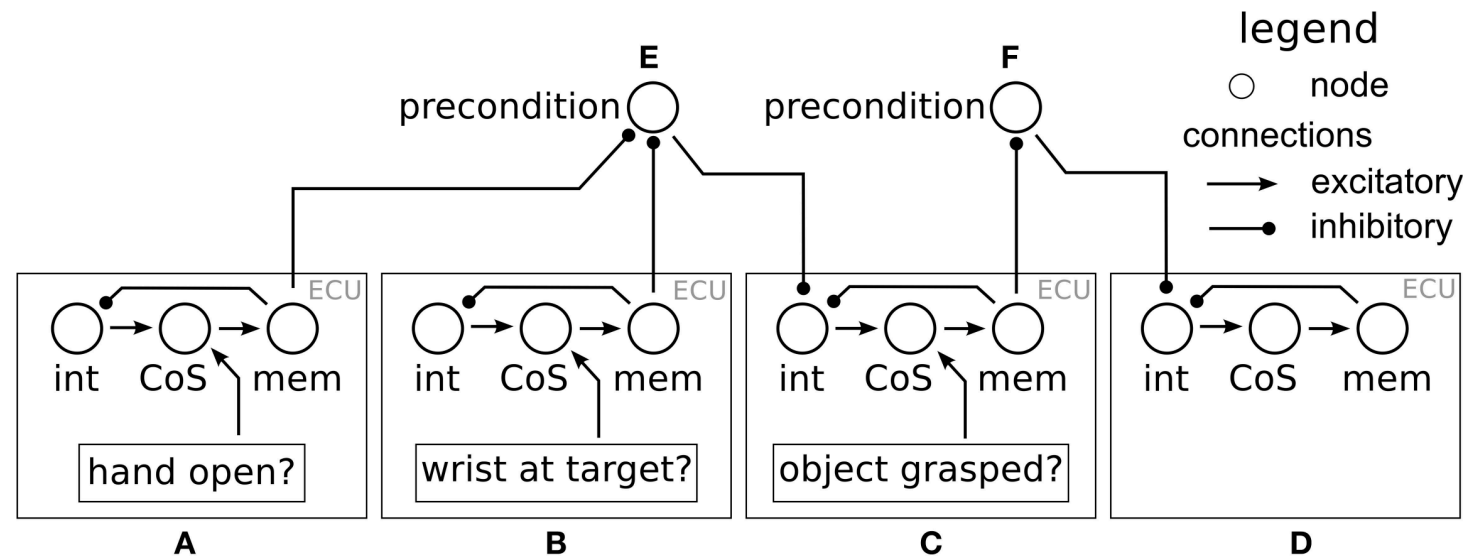
- planning sequence of movements toward goal
- extracting movement parameters for each movement



[Schöner, Tekülve, Zibner, 2019]

# Movement organization

- initiation and termination of each movement
- serial/parallel activation of different movement “primitives”



# Trajectory formation

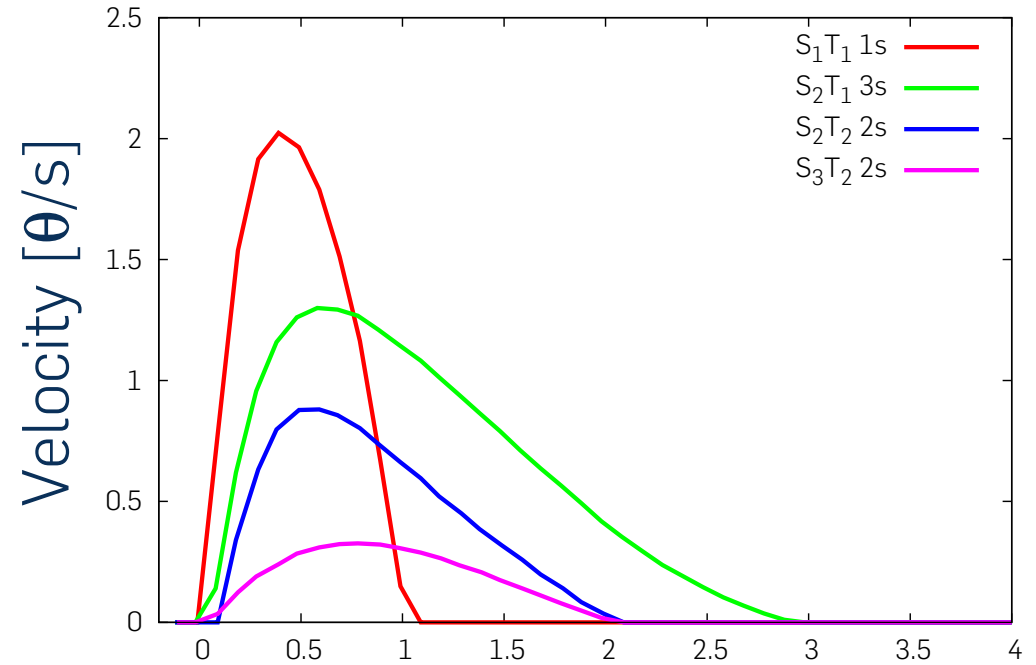
- generating end-effector velocity reference command

- here: from a field of neural oscillators

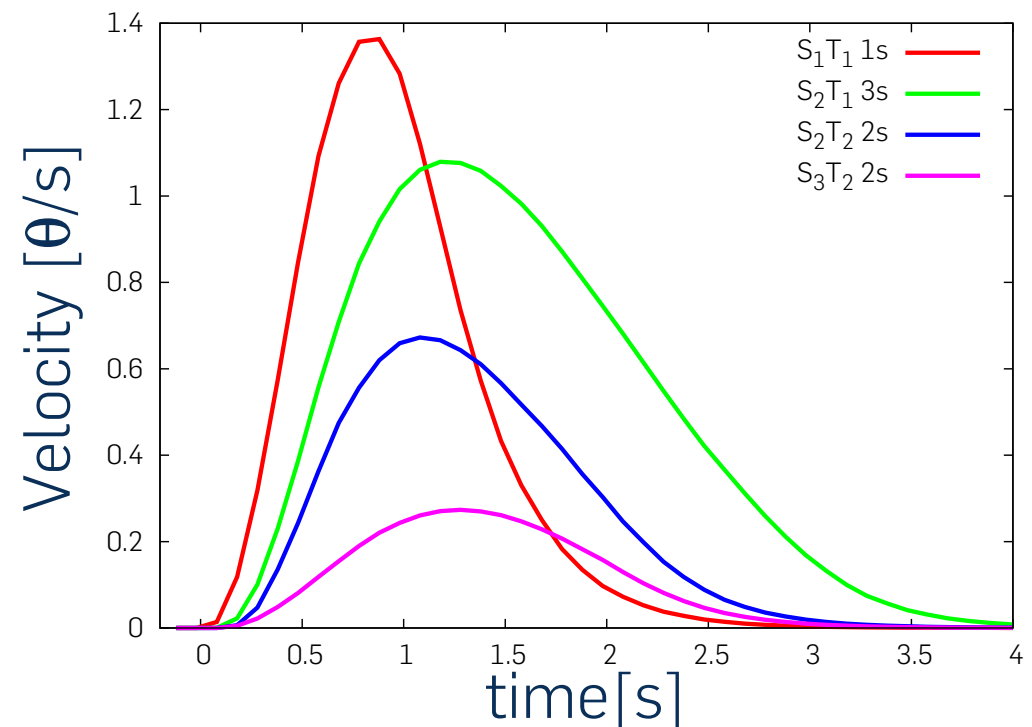
=> lecture on timing and coordination

[Zibner, Tekülve, Schöner, ICDL 2015; Schöner, Tekülve, Zibner, 2019]

virtual velocity from oscillator



hand velocity



# Control

■ bringing about the physical movement...

=> lecture on control

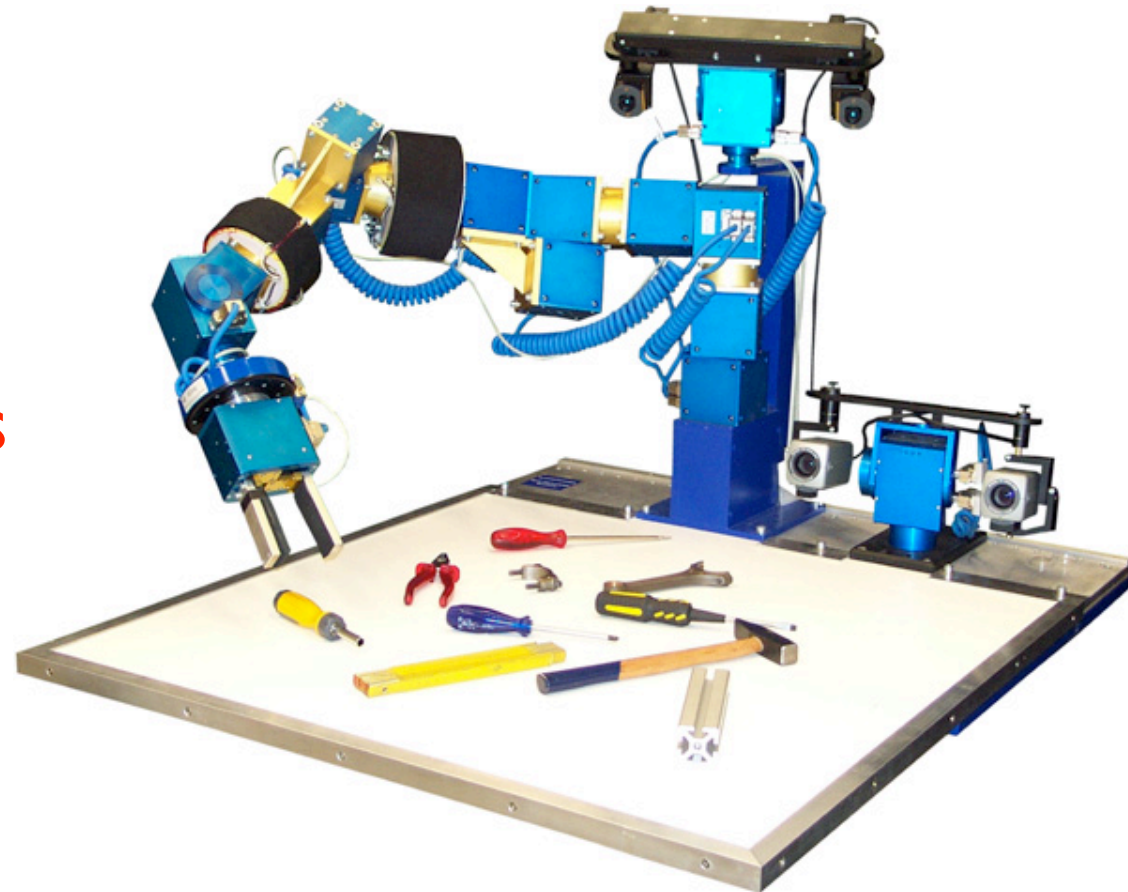
# Basic concepts for robotic arms

- task vs control level: degree of freedom problem
- rigid body motion
- kinematics vs kinetics
- kinematic chain
- manipulator kinematics
- redundant manipulator kinematics



# Levels of movement planning/control: Degree of freedom problem

- Perception, planning, organization, and timing are about the **task level**
- but control is at the **level of the manipulator's actuators...**



# Task level

- target state at task level

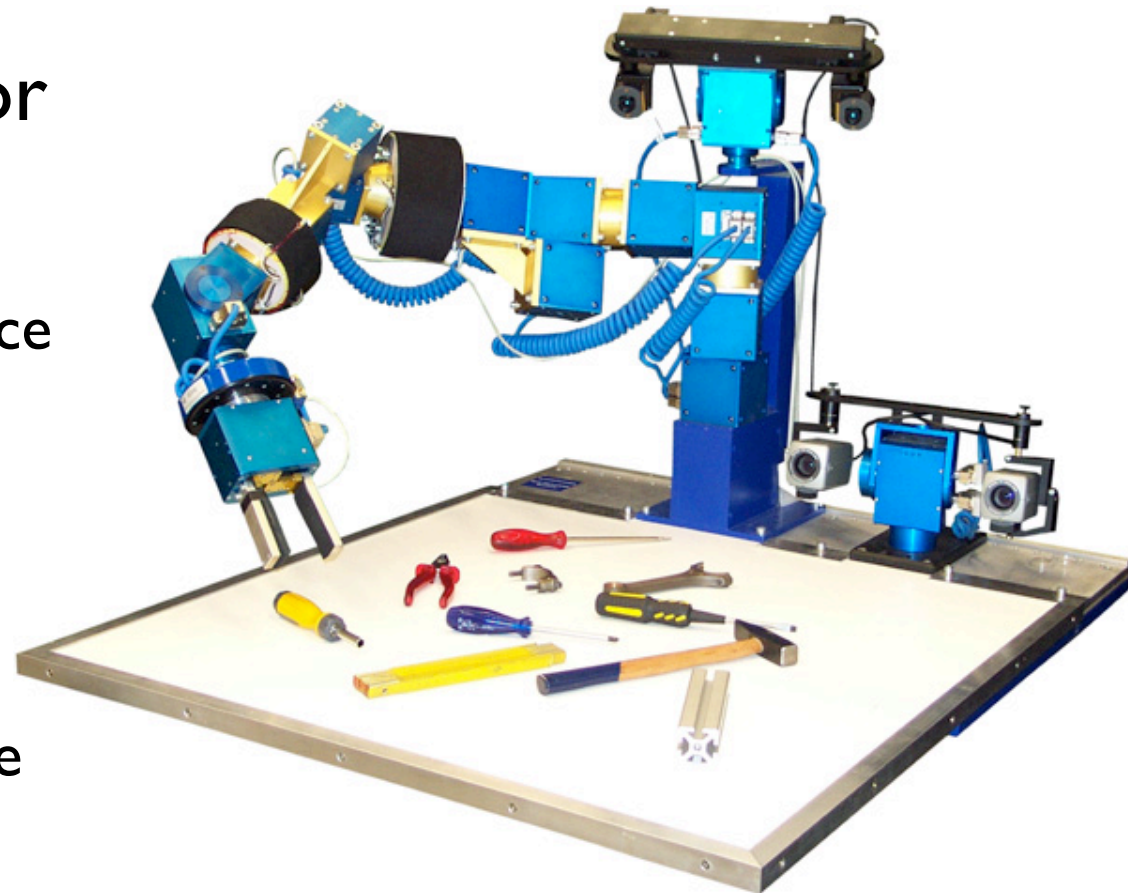
- 3D position of gripper/hand and 3D orientation of gripper/hand

- other task constraints for other task variables

- e.g. closes point on arm surface to an obstacle

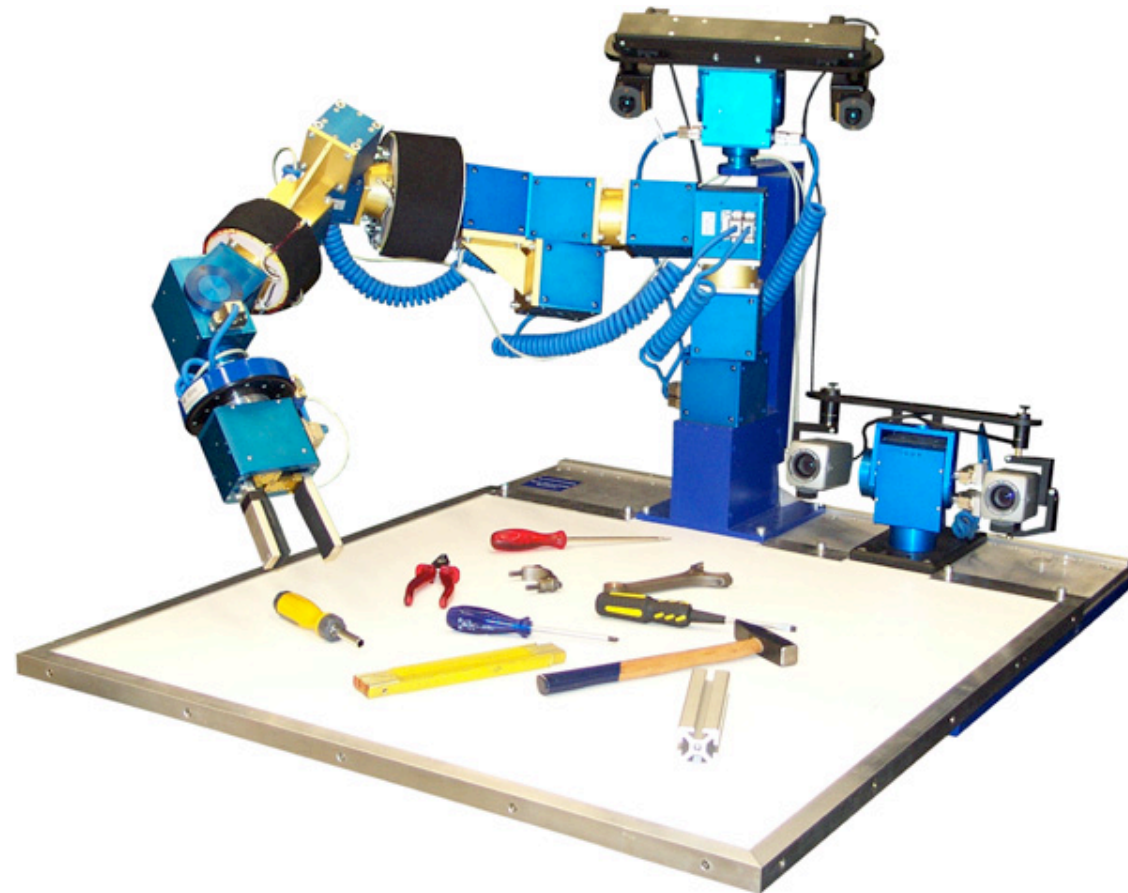
- timing at task level

- e.g. for catching, hand at the right position at the right time



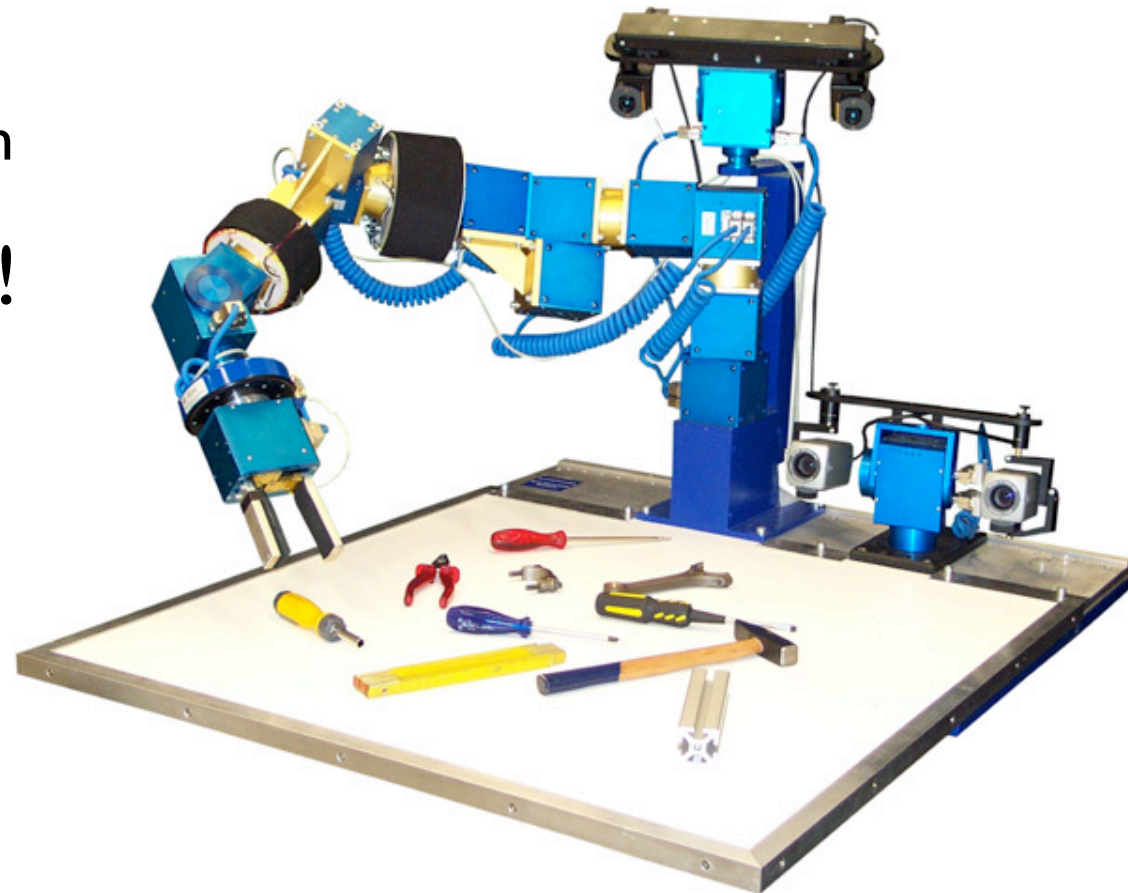
# Control level

- mechanical degrees of freedom...
  - e.g. joint angles
- at which actuation takes place
  - motors
  - spring pre-load/muscle activation levels



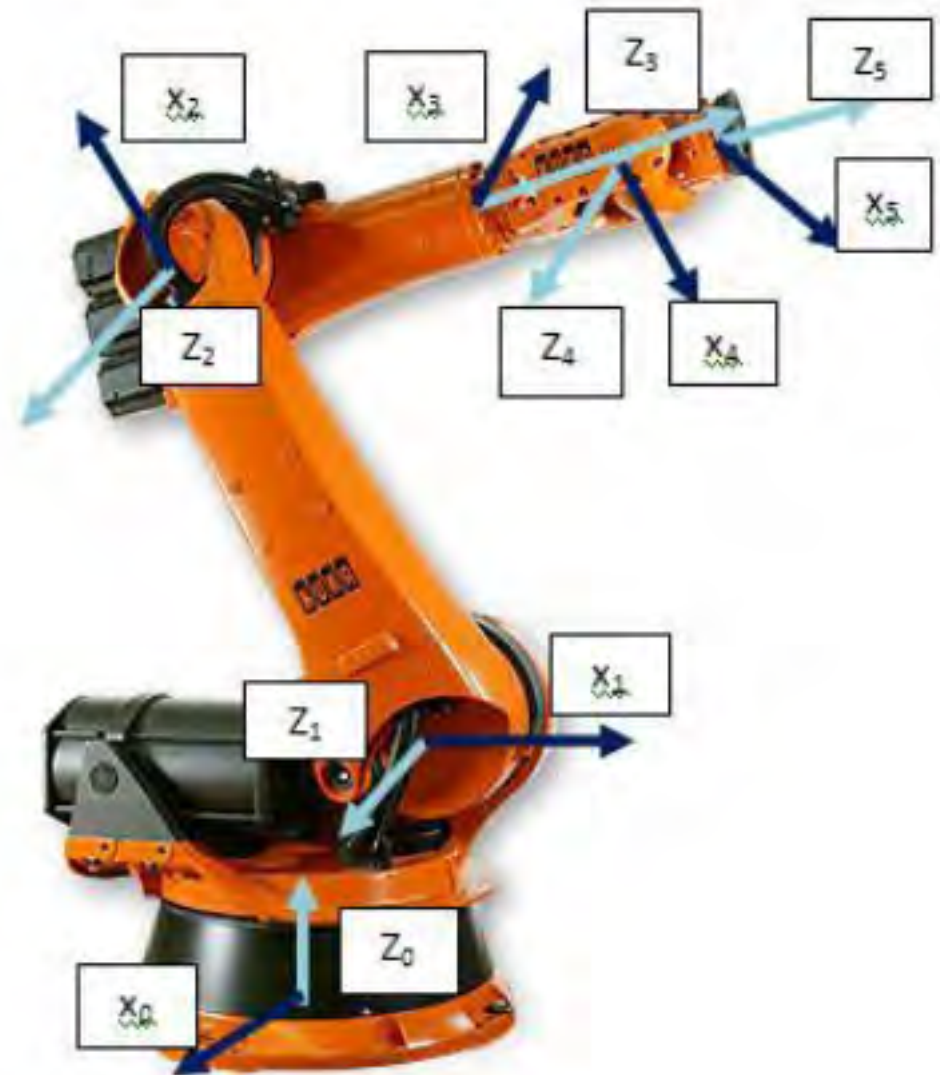
# Redundancy

- when there are more variables at the control than the task level
  - e.g. 10 joints for human-like arm vs 3+3 coordinates for hand position and orientation
- => depends on the task!
- that gap between task and control level is the “degree of freedom problem”



# Degree of freedom problem

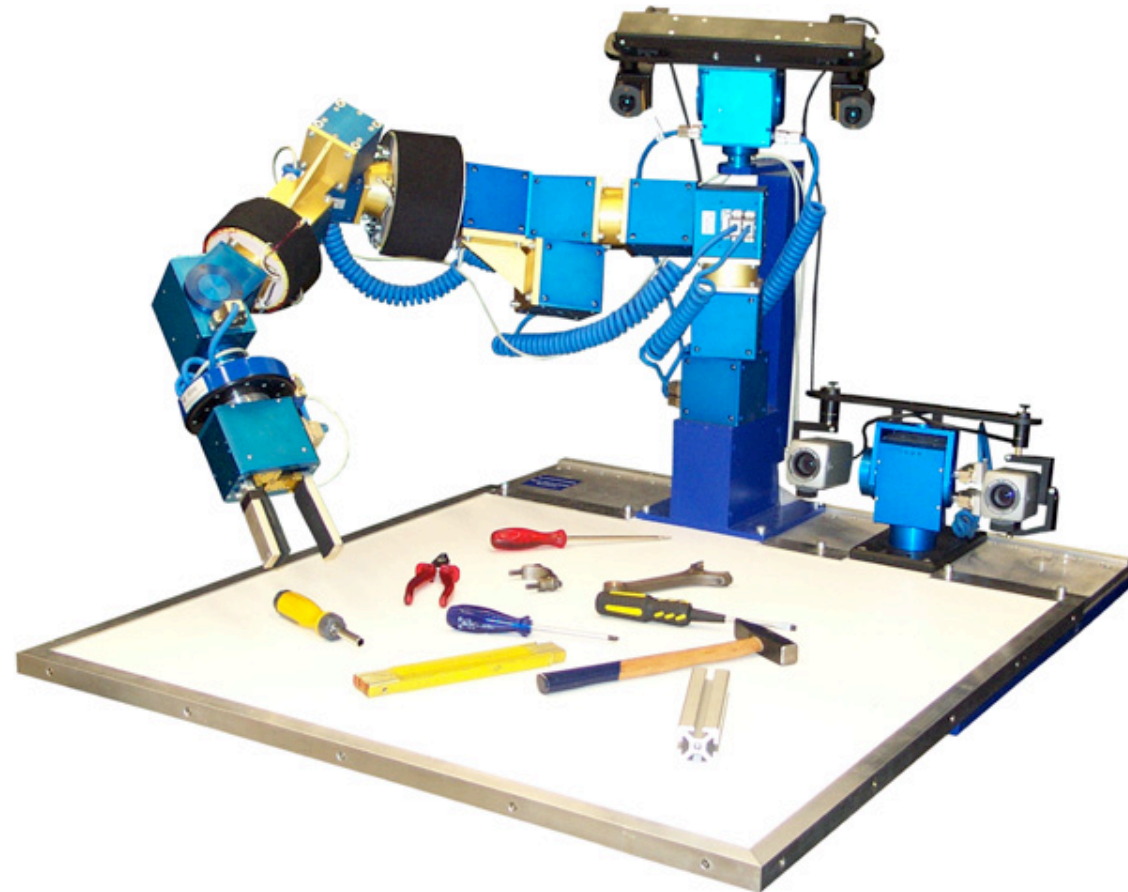
- many conventional robot arms are not redundant for end-effector task
- most commonly: 3+3 hand/gripper task variables and 6 actuated joints



[Kuka KR 16KS: Dahari, Tan 2012]]

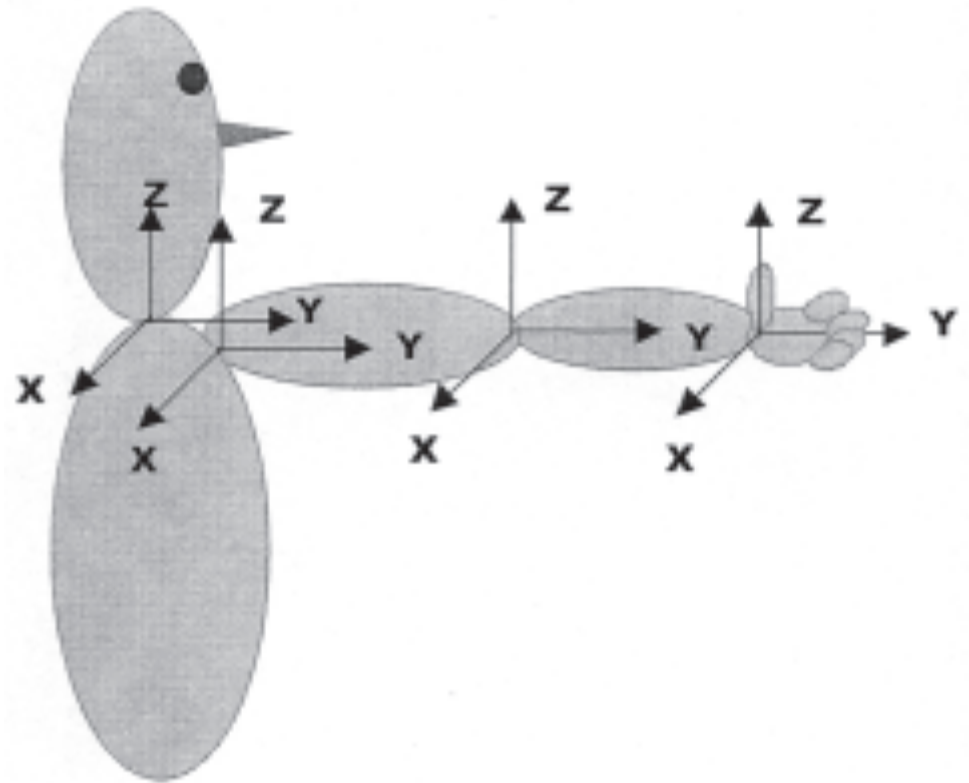
# Degree of freedom problem

- but: some manipulators are redundant for some tasks
- which gives them added flexibility across tasks
- or enables them to deal with multiple tasks at the same time



# Degree of freedom problem

- The human motor system is redundant for many tasks
- e.g. upper extremity for reaching/pointing
- $> 10$  Dof
- ca. 40 muscles
- 3-6 hand pose task variables



[Tseng, Scholz, Schöner, 2002]

# Basic concepts

- task vs control level: degree of freedom problem

- rigid body motion

- kinematics vs kinetics

- kinematic chain

- manipulator kinematics

- redundant manipulator kinematics

conventional  
robotics



■ Murray, Li, Sastry: A Mathematical Introduction to Robotic Manipulation, CRC Press, Boca Raton FL USA 1994

■ a pdf is made available by the authors

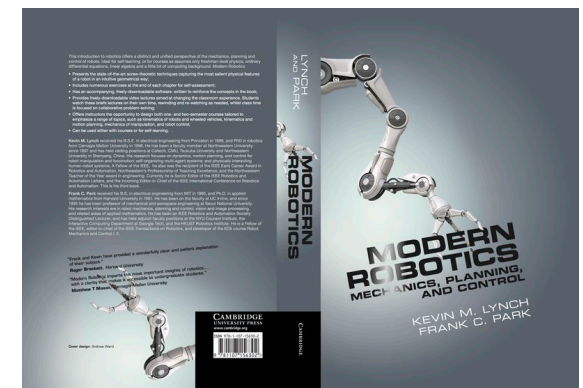
■ quite an advanced text

■ Lynch, Park: Modern robotics— mechanics, planning, and control. Cambridge Univ. Press, 2017

■ pre-print version available online from others

■ a more tutorial text

MODERN ROBOTICS  
MECHANICS, PLANNING, AND CONTROL



KEVIN M. LYNCH AND FRANK C. PARK

[Murray, Li, Sastry, 1994]

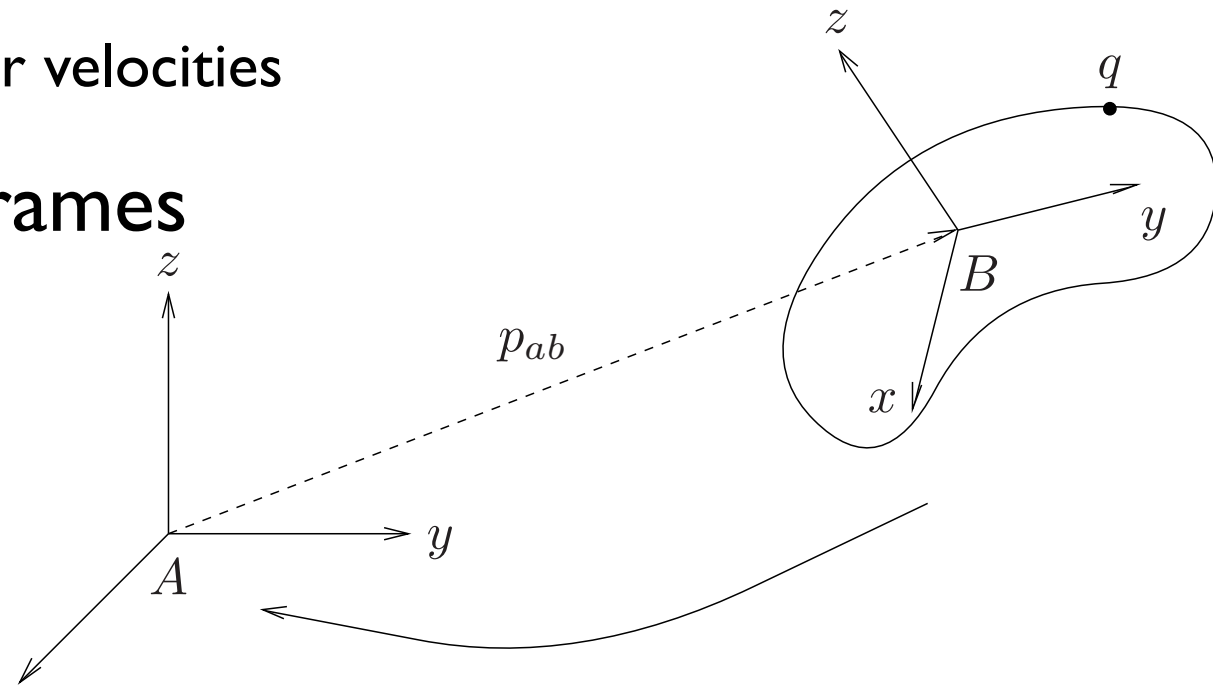
# Rigid body motion

- a rigid body performs motion in 6D

- three positions, three orientations

- three linear, three angular velocities

- SE(3) transforming frames



- description of such motion by

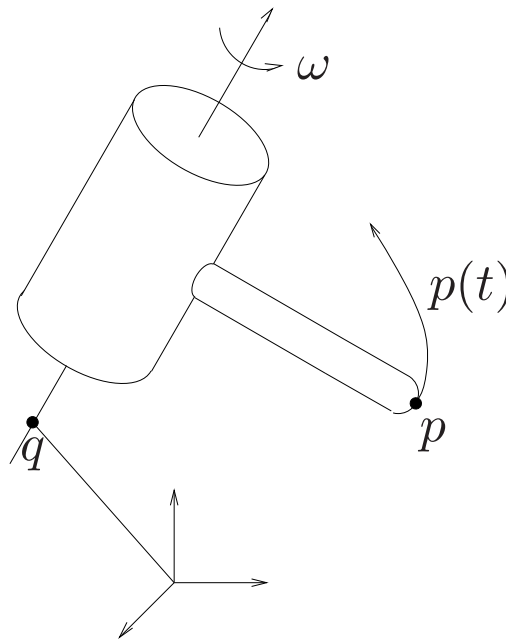
[Murray, Li, Sastry, 1994]

- the position vector

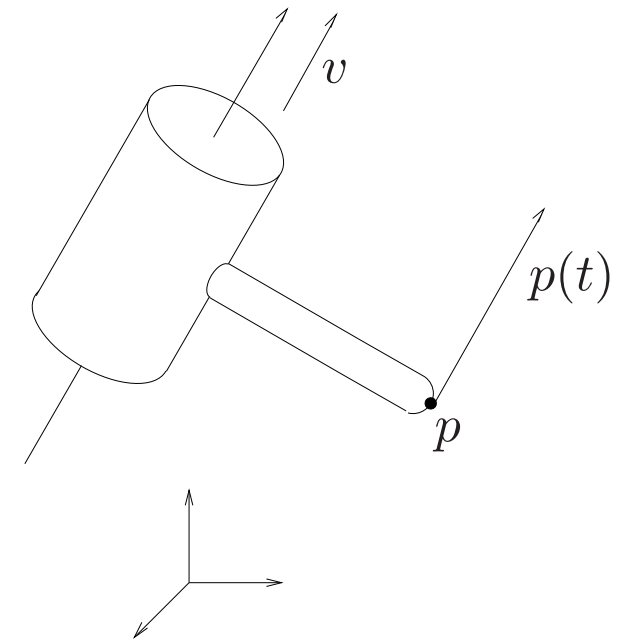
- a representation of rotation (Euler angles, Rotation matrix, generator of Lie group)

# Rigid body motion

- constraints... revolute, prismatic, spherical.. joints
- reduce the number of degrees of freedom
- holonomic: can be formalized by reducing the number of variables



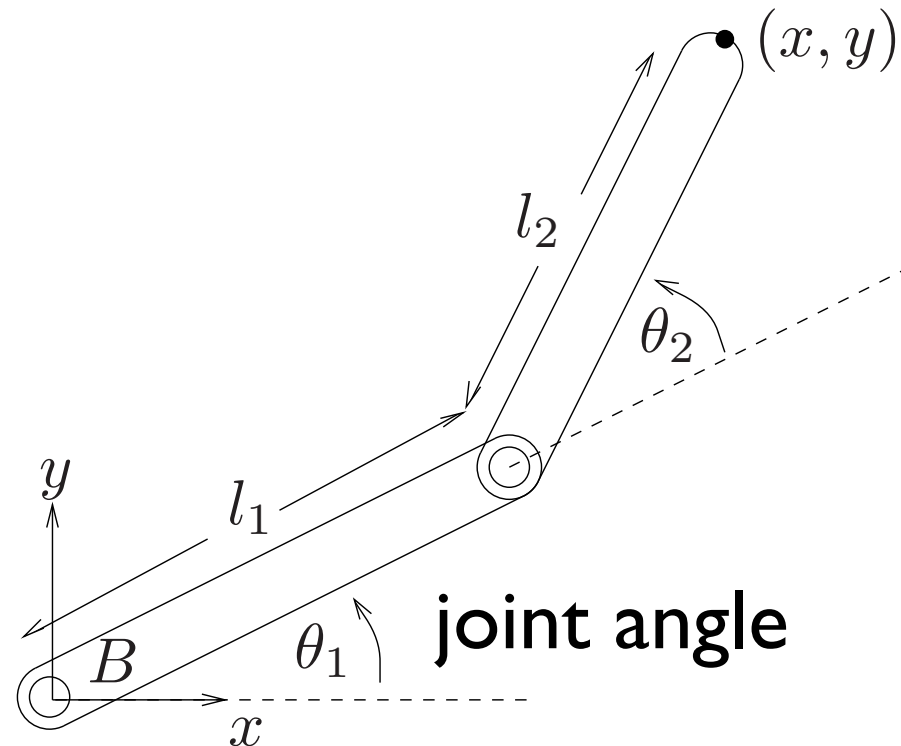
revolute joint



prismatic joint

# Rigid body motion

- in a in a kinematic chain, the degrees of freedom of each rigid segment is reduced
- for revolute or prismatic joints to a single(!) degree of freedom captured



[Murray, Li, Sastry, 1994]

# Kinematics vs Kinetics

- kinematics: the description of the possible spatial (and velocity space) configurations of an arm taking into account the constraints

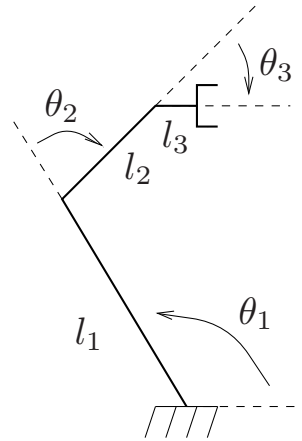
- treated now

- kinetics: the dynamic equations of motion of an arm taking into account the constraints, gravity, and actuators mounted on the joints

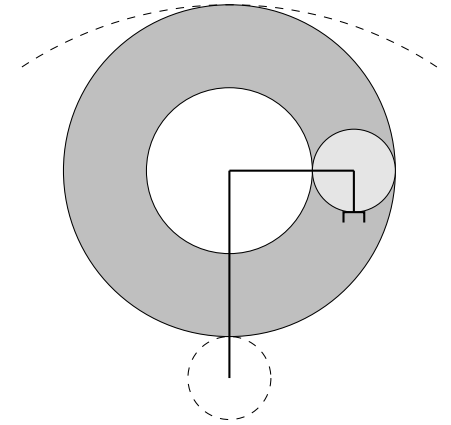
- (later in the lecture series)

# Kinematic chain

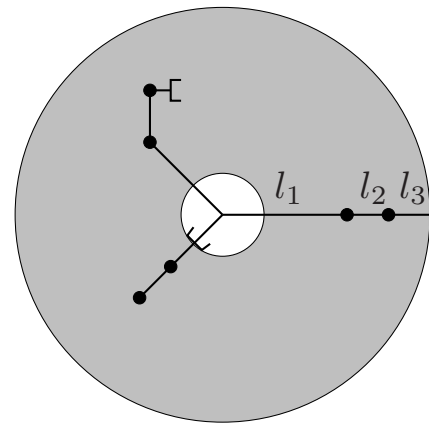
■ notion of work space



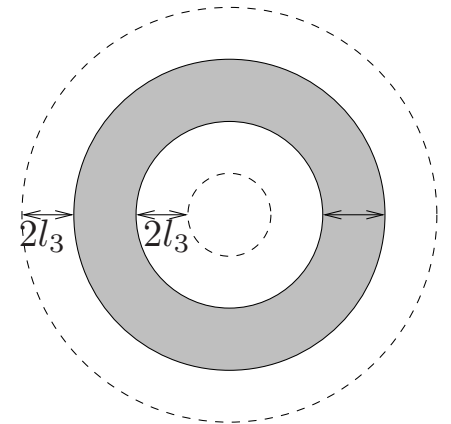
(a)



(b)



reachable space



dexterous space:  
reachable with  
arbitrary orientation

[Murray, Li, Sastry, 1994]

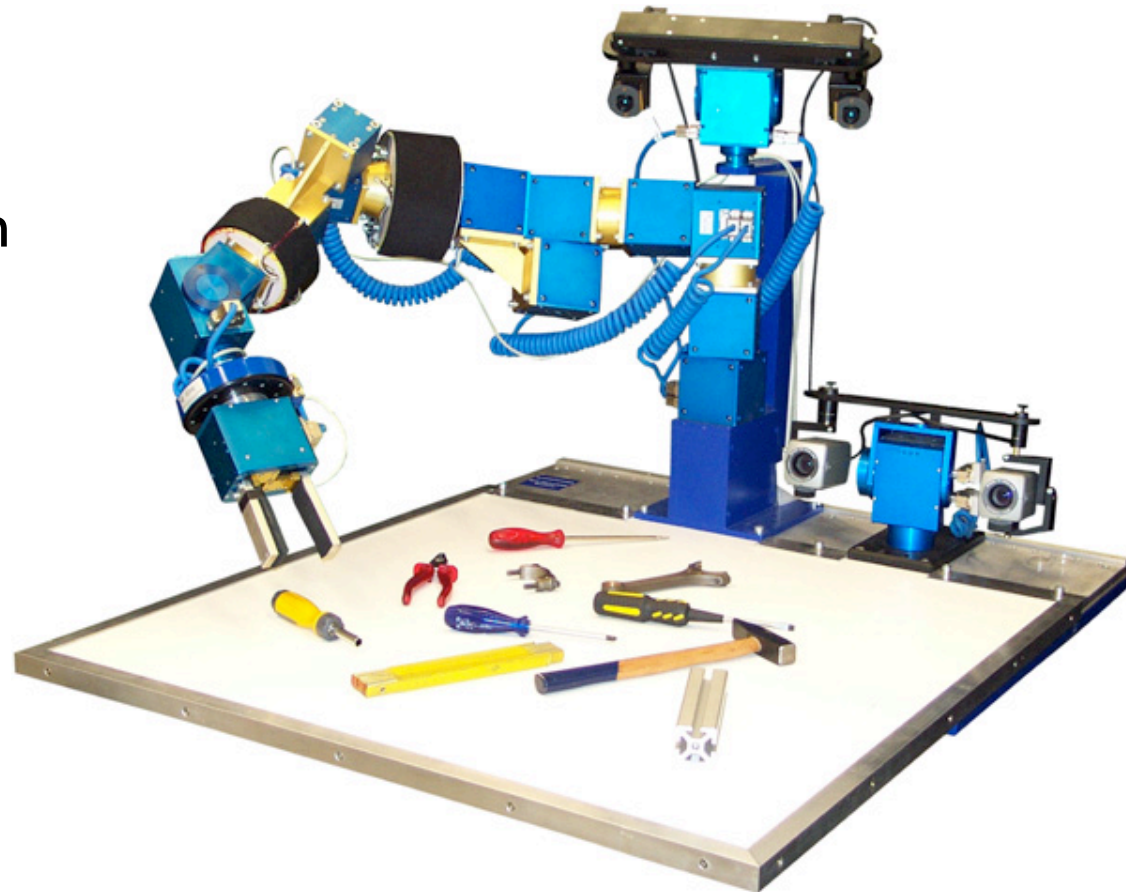
# Manipulator kinematics

■ end-effector

■ e.g. with 3 translational and 3 rotational degrees of freedom

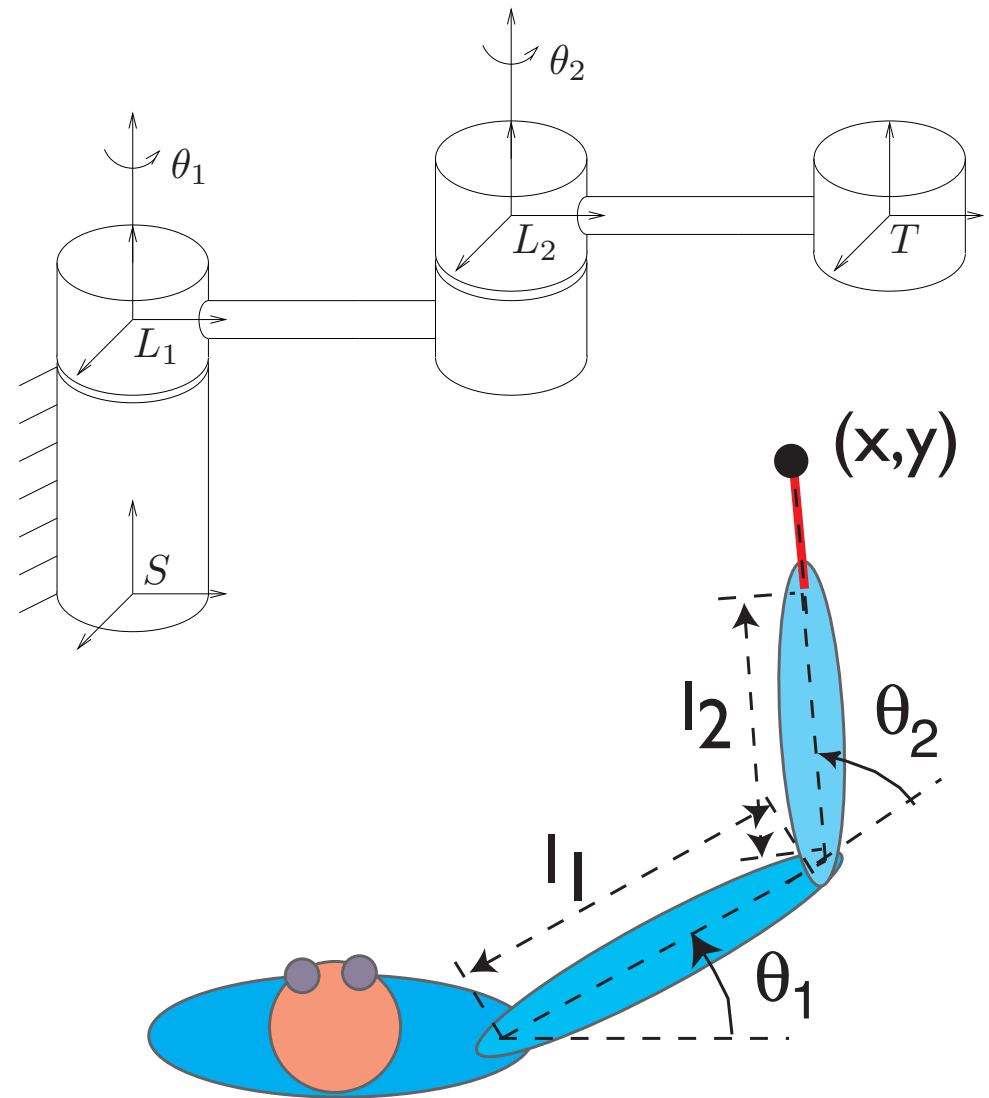
■ configuration space

■ e.g. 7 actuated joint angles



# Forward kinematics

[Murray, Li, Sastry | 1994]



- where is the hand, given the joint angles..

$$\mathbf{x} = \mathbf{f}(\theta)$$

$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2)$$

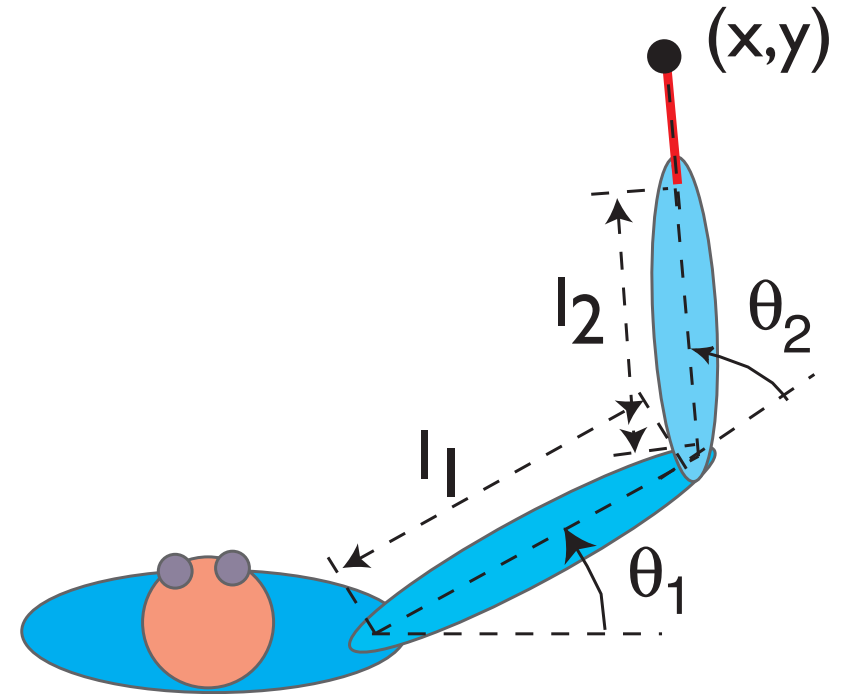
$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2)$$



# Differential forward kinematics

- where is the hand moving, given the joint angles and velocities

$$\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$$

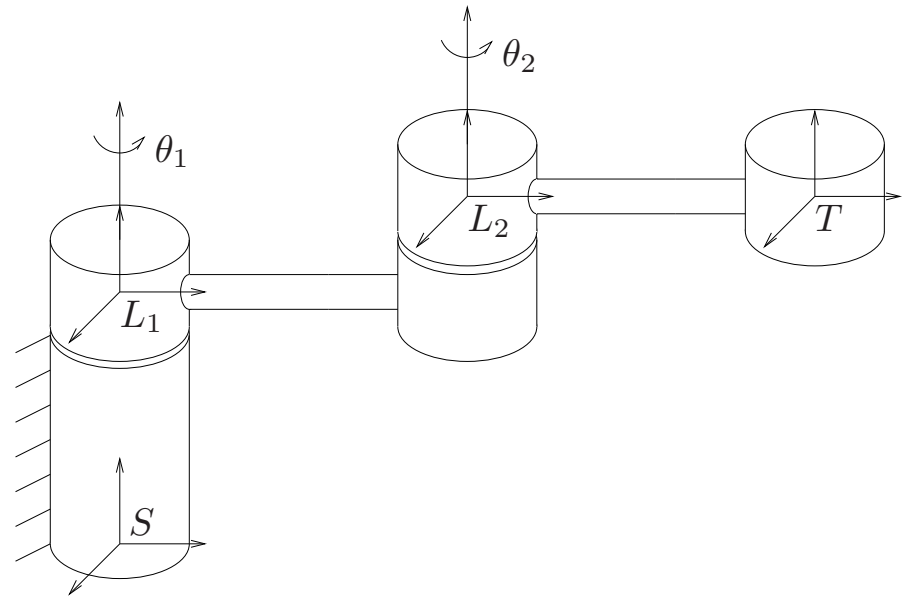


$$\dot{x} = -l_1 \sin(\theta_1)\dot{\theta}_1 - l_2 \sin(\theta_1 + \theta_2)\dot{\theta}_1 - l_2 \sin(\theta_1 + \theta_2)\dot{\theta}_2$$

$$\dot{y} = l_1 \cos(\theta_1)\dot{\theta}_1 + l_2 \cos(\theta_1 + \theta_2)\dot{\theta}_1 + l_2 \cos(\theta_1 + \theta_2)\dot{\theta}_2$$

# Formal approach to kinematics

- base frame  $S$  and tool frame  $T$
- ask how any object represented in the base frame is represented in the tool frame  $\Rightarrow$  forward kinematics
- transformation from base frame  $g_t$  to tool frame  $g_s$  is a function of the joint angles

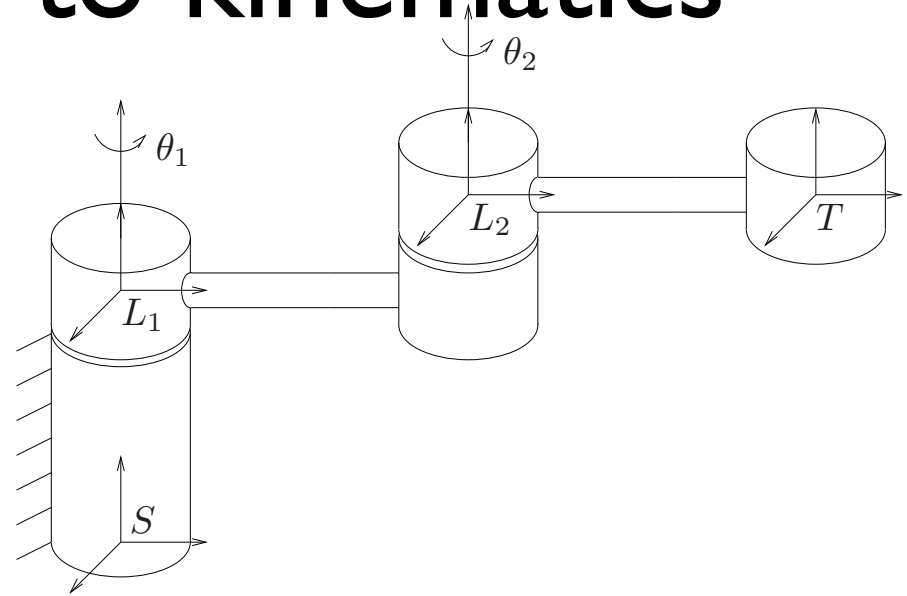


$$g_{st}(\theta_1, \theta_2) = g_{sl_1}(\theta_1)g_{l_1l_2}(\theta_2)g_{l_2t}.$$

# Formal approach to kinematics

## ■ product of exponentials

$$g_{st}(\theta_1, \theta_2) = g_{sl_1}(\theta_1)g_{l_1l_2}(\theta_2)g_{l_2t}.$$



$$g_{st}(\theta_1, \theta_2) = e^{\hat{\xi}_1\theta_1}g_{st}(\theta_2) = e^{\hat{\xi}_1\theta_1}e^{\hat{\xi}_2\theta_2}g_{st}(0),$$

$$e^{\hat{\xi}_1\theta_1} = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$e^{\hat{\xi}_2\theta_2} = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & l_1 \sin \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & l_1(1 - \cos \theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

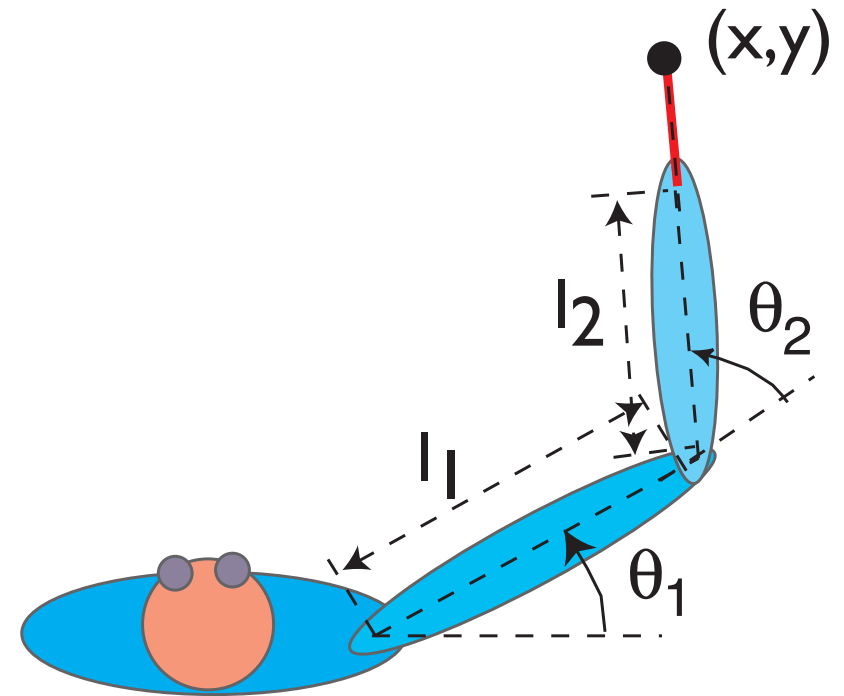
$$\xi_i = \begin{bmatrix} -\omega_i \times q_i \\ \omega_i \end{bmatrix},$$

$$g_{st}(0) = \begin{bmatrix} I & \begin{pmatrix} 0 \\ l_1+l_2 \\ l_0 \end{pmatrix} \\ 0 & 1 \end{bmatrix}.$$

# Inverse kinematics

- what joint angles are needed to put the hand at a given location
- exact solution:

$$\theta = \mathbf{f}^{-1}(\mathbf{x})$$



# Inverse kinematics

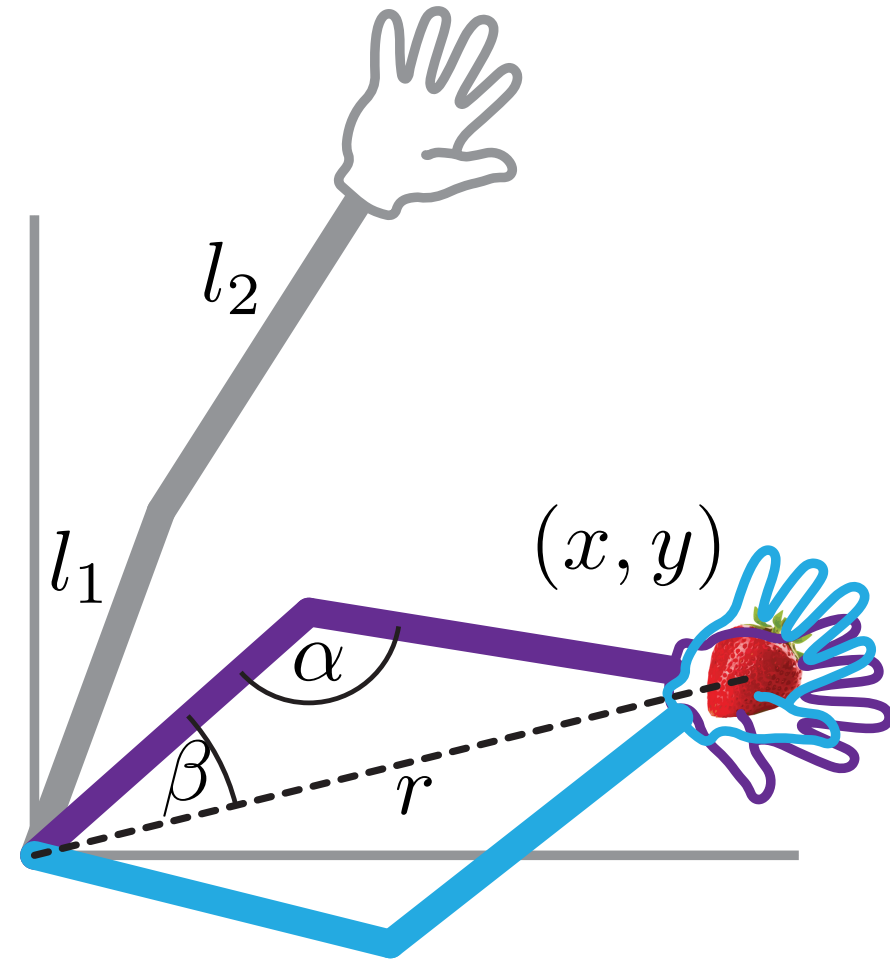
$$\theta_1 = \arctan_2(y, x) \pm \beta$$

$$\theta_2 = \pi \pm \alpha$$

$$\alpha = \cos^{-1} \left( \frac{l_1^2 + l_2^2 - r^2}{2l_1l_2} \right)$$

$$\beta = \cos^{-1} \left( \frac{r^2 + l_1^2 - l_2^2}{2l_1l_2} \right)$$

where  $r^2 = x^2 + y^2$



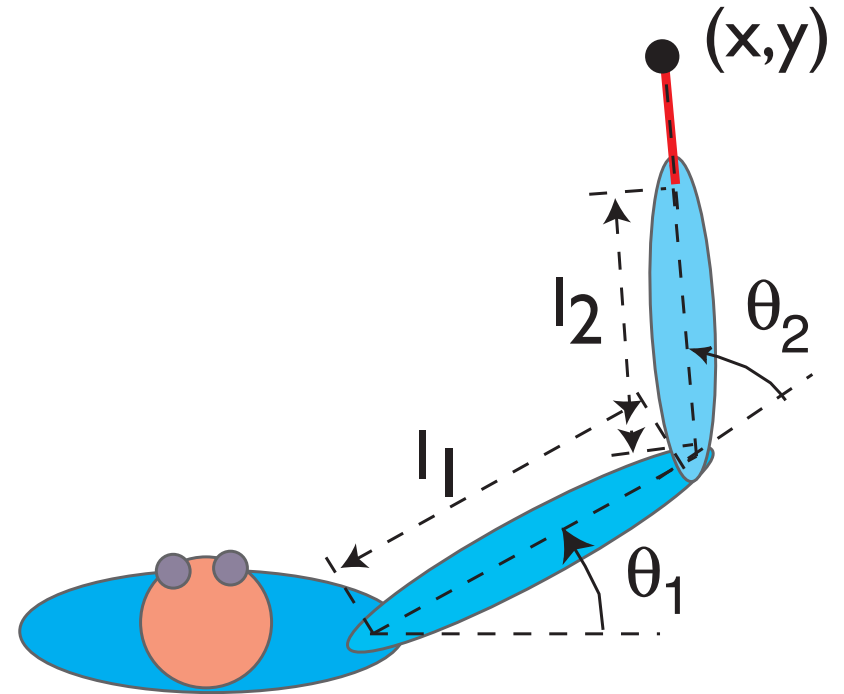
[thanks to Jean-Stéphane Jokeit]

■ => multiple “leafs” of the inverse kinematics

# Differential inverse kinematics

- which joint velocities to move the hand in a particular way

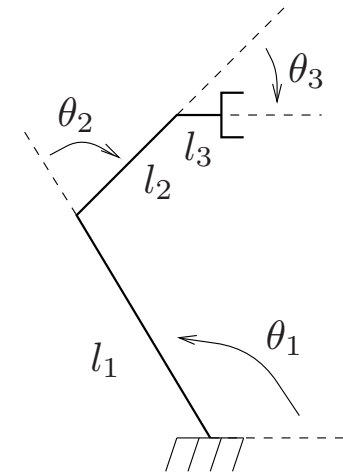
$$\dot{\theta} = \mathbf{J}^{-1}(\theta)\dot{\mathbf{x}}$$



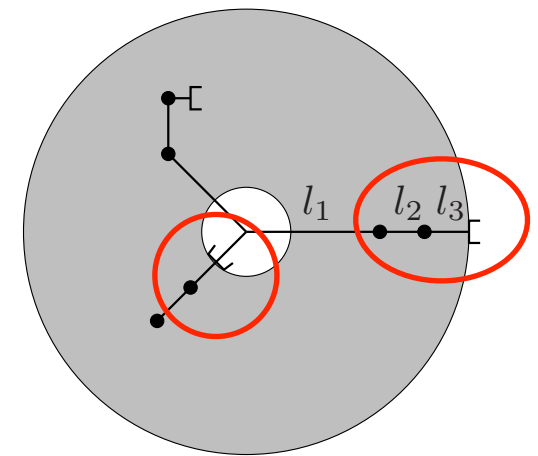
with the inverse,  $\mathbf{J}^{-1}$ , of  $\mathbf{J}$ , if it exists

# Singularities

- where the Eigenvalue of the Jacobian becomes zero (real part)...
- so that movement in a particular direction is not possible...
- typically at extended postures or inverted postures
- at limit of workspace



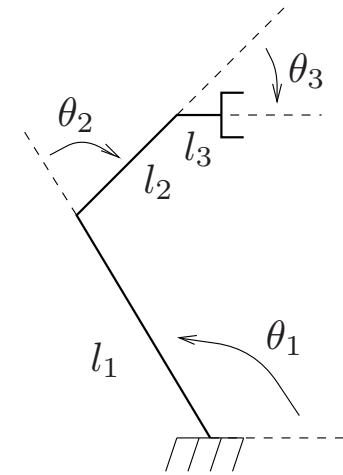
(a)



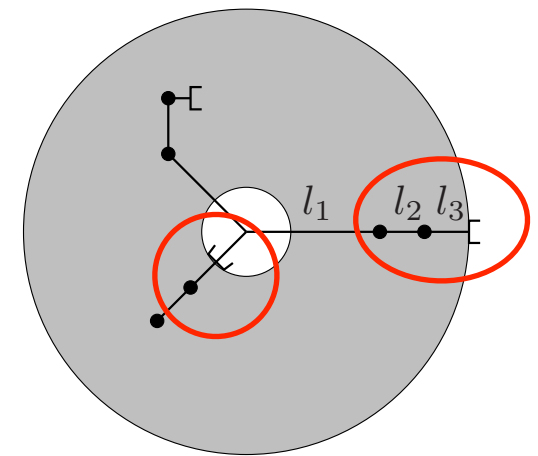
(c)

# Singularities

- leading to non-invertability!
- and to sensitive dependence on parameters
- => avoid singularities in motor planning... major effort in robotics
- humans: joint angles prevent us from getting near singularities (for the most part)



(a)



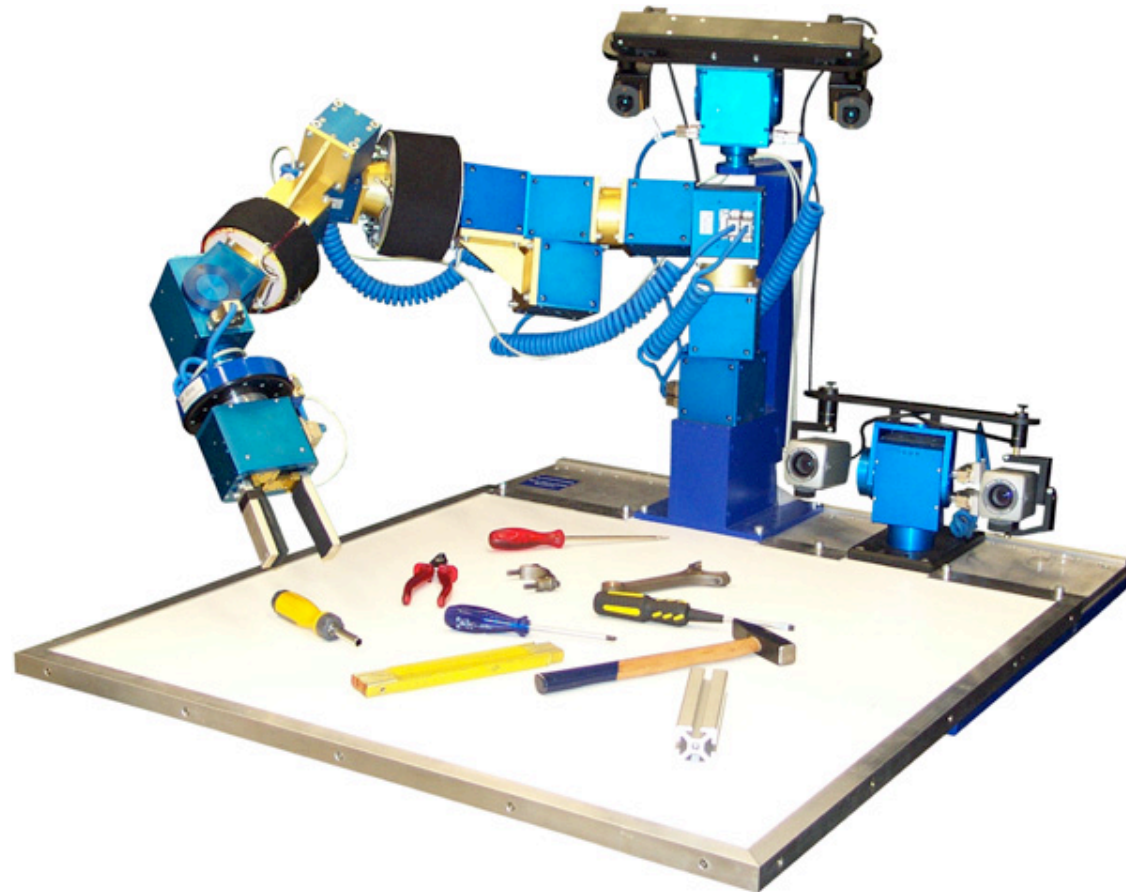
(c)



# Summary arm kinematics

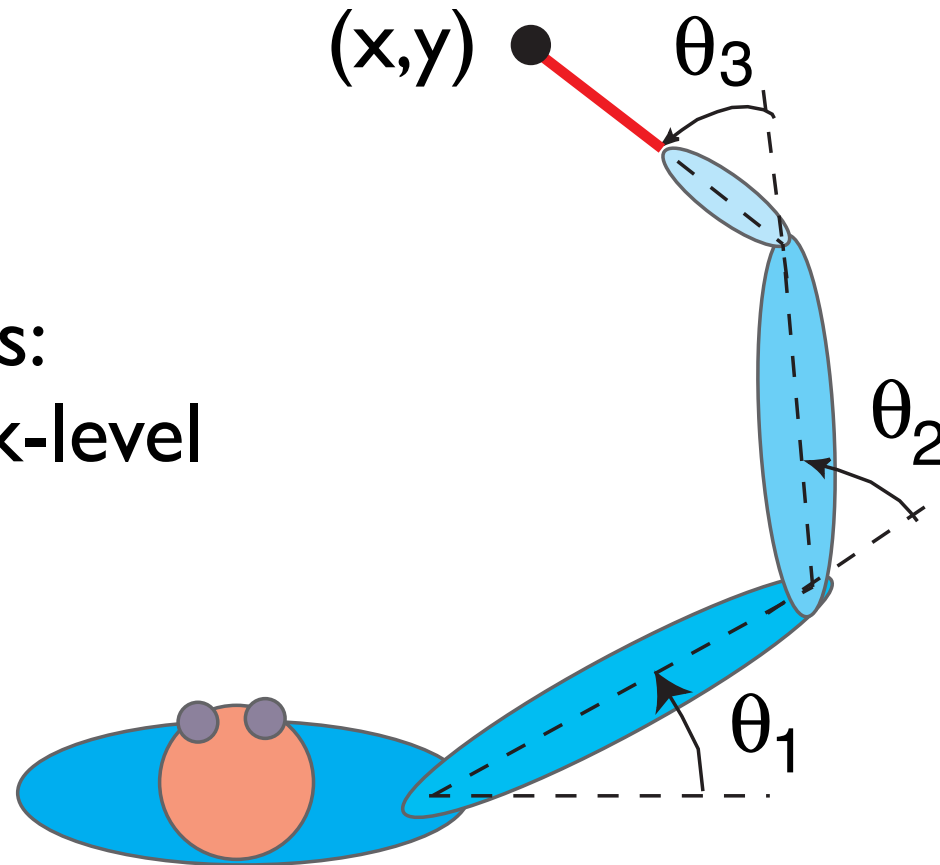
kinematic model  $\mathbf{x} = \mathbf{f}(\theta)$   $\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$

inverse kinematic model  $\theta = \mathbf{f}^{-1}(\mathbf{x})$   $\dot{\theta} = \mathbf{J}^{-1}(\theta)\dot{\mathbf{x}}$



# Redundant kinematics

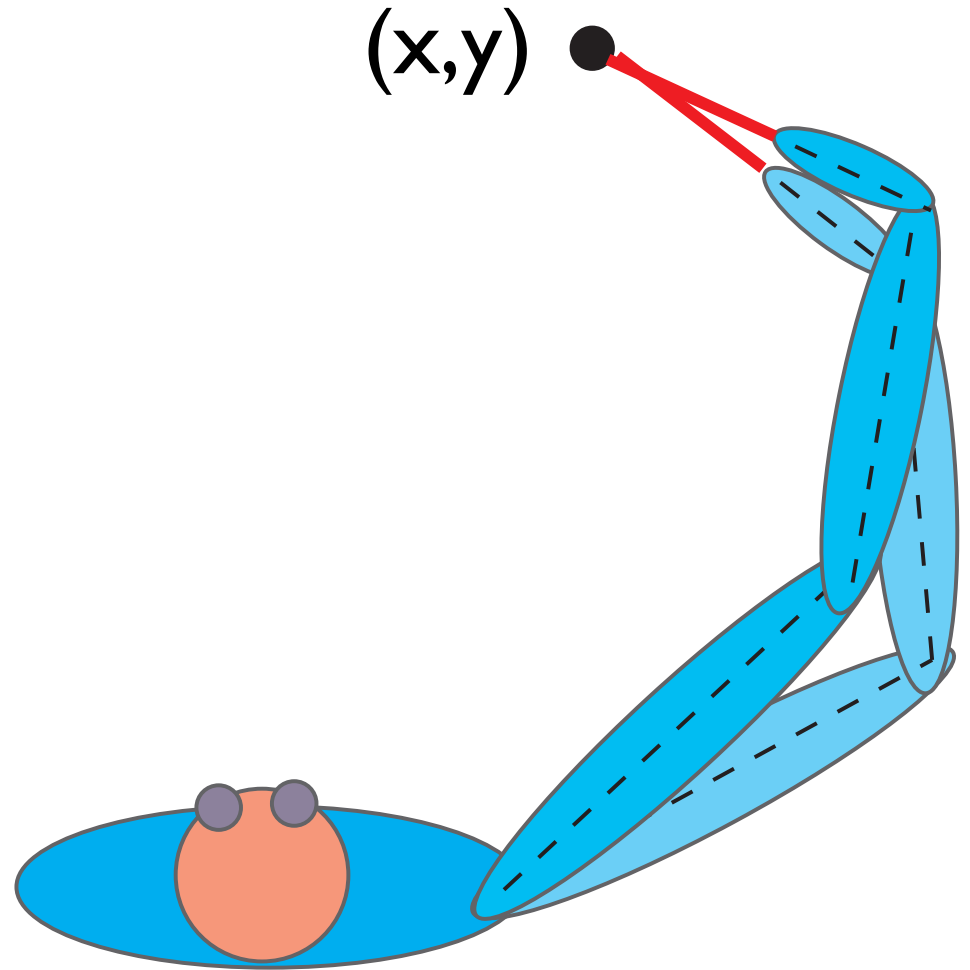
- redundant arms/tasks:  
more joints than task-level  
degrees of freedom



$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

# Redundant kinematics

■ => (continuously) many  
inverse solutions...



# Redundant kinematics

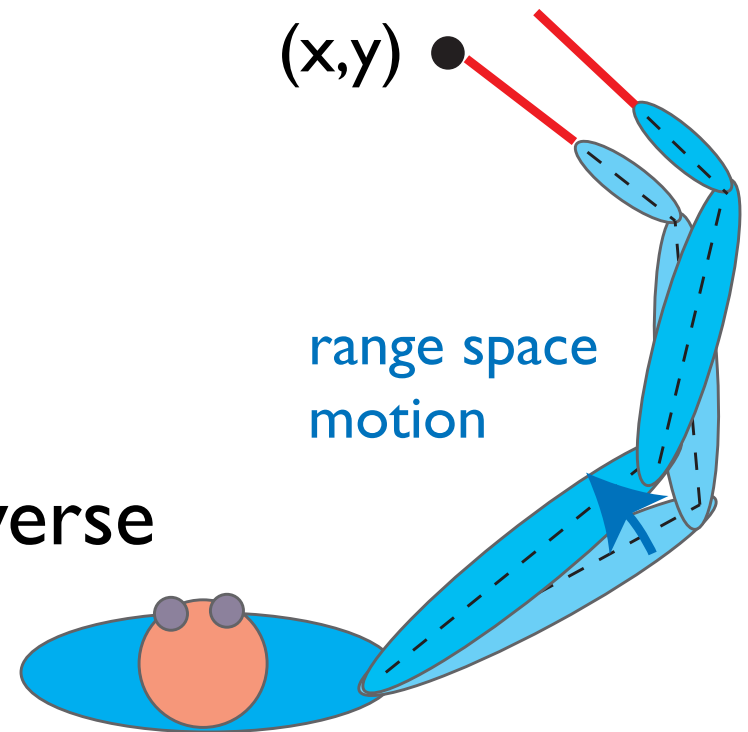
- use pseudo-inverses that minimize a functional (e.g., total joint velocity or total momentum)

$$\dot{\mathbf{x}} = \mathbf{J}(\theta)\dot{\theta}$$

$$\dot{\theta} = \mathbf{J}^+(\theta)\dot{\mathbf{x}}$$

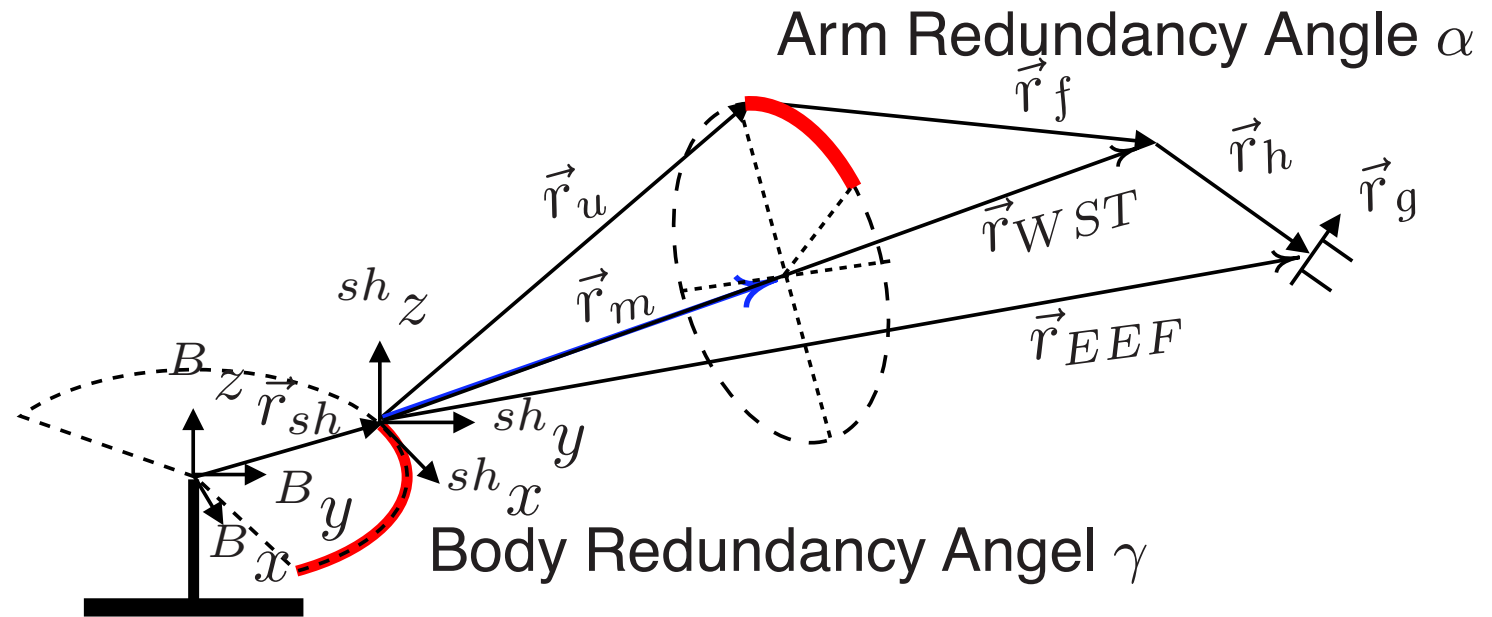
$$\mathbf{J}^+(\theta) = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1} \quad \text{pseudo-inverse}$$

minimizes  $\dot{\theta}^2$



# Spaces for robotic motion planning

- or use extra degrees of freedom for additional tasks



[Iossifidis, Schöner, ICRA 2004]

# Basic concepts

- degree of freedom problem
- rigid body motion
- kinematics vs kinetics
- kinematic chain
- manipulator kinematics
- redundant manipulator kinematics